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Analytical Solution of Some Non – Linear Delay Differential Equations Using Adomian Decomposition Method

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ABSTRACT

This study examined some Non-linear Delay Differential Equation problems with known exact solutions and solved them using the Adomian Decomposition Method. We then observed the results and compared them with the corresponding exact solutions, and ultimately found that the two results agree. In addition, we discussed the phenomena that occur in real life and are described by Non-linear Differential Equations.

Keywords : Non-Linear Differential Equation, Delay Differential Equation (DDE), Adomian Decomposition Method (ADM).

I. INTRODUCTION

American mathematician George Adomian lived from 1922 to 1996 [1]. He developed the extensive analytical technique known as the Adomian decomposition method, which divides equations into linear and non-linear equations. [2] The general solution assumes a series form, allowing the Adomian Polynomial to be used to generate a recursive formula[6].

The Adomian Decomposition Method is an analytical technique used to solve differential equations in a variety of ways. It provides a quick convergent series solution and may be used to different differential equations to produce a closed form and various numerical approximations [3]. The underlying theory behind these delay differential equations (DDE) is similar to that of ordinary differential equations (ODE), although they can occasionally be challenging analytically due to non-linearity or time-varying coefficients. DDE is mostly utilized as a crucial mathematical model technique in problem reduction across various fields. DDE is only the result of a delay system where the unknown function at a given time is determined by the value of the function at a previous time.[4]

Linear Differential Equation: In this form, K and L are functions within V. It is made up of U and a derivative of U.

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Differential Equation (Linear): The form $\frac{du}{dV} + ku = L$ where K and L are functions in V. it comprises of u and a derivative of u.

Differential Equation (Non-linear): A non-linear form is one that is not linear in an undefined function.[6]

Delay Differential Equation: Time delay systems use this equation to determine the unknown function at a particular time based on the value of the function at a previous time. [9]

Closed Form: An equation represented in terms of a restricted or limited function [13]

Differential Equation: An equation whose derivative contains one or more undefined variables [4].

II. METHODS

Let L^{-1} be the inverse operator of Delay Differential Equation L such that

$L(*) = \frac{d^{ll}(*)}{dx^{ll}}$

$L^{-1}(\$) = \int_0^x (\$)_{N \text{ times}} dx$

Given Delay Differential Equation, apply L^{-1}

$L(y(x)) = f(x, y(x), y(h(x)))$

Write like the

$L(y(x)) = f(x, y(x))$, where

$y(x) = y(h(x)). \quad x \in [0, 1]$

$y^{(K)}(0) = y_0^k$, $K = 0,1,2, \ldots, n-1$

$y(x) = ß(x) \qquad x \leq 0$

then

$y(x) = \sum_{j=0}^{N-1} \frac{ß}{j} x^j + L^{-1} f(x, y(x), y(h(x)))$,

Where $ß_j$ is constant coefficient that describe the boundary conditions. Adomian decomposition method assumes that the undefine function y(x) can describe by

$y(x) = \sum_{j=0}^{N-1} y_n(x)$

Adomian polynomials is generated

$f(x, y(x), h(y(x))) = = \sum_{n=0}^{\infty} A_n(x)$,

III. RESULTS AND DISCUSSION

Problem 1:

$$xz^{lll} - z - xz^{l} = 0 \qquad \{1\}$$

given $z(0) = 0$, $z^{1}(0) = 1$ and $z^{ll}(0) = 2$

Where the recursive relation is given as

$$f_{i,n+1} = \int_0^x A_{i,n}\left(f_{i,0,\ldots,} f_{i,n}\right) dx \qquad \{2\}$$

let

$z_1^l = z_2 \;\; z_2^l = z_3 \;\; and \; z_3^l = \frac{1}{x} z_1 + z_3$

if we apply the inverse operator and use the alternate algorithm for computing Adomian Polynomials, we would have the following.

$Z_{1,0} = 0 \quad Z_{1,n+1} = \int_0^x (Z_{2,n}) dx$

$Z_{2,0} = 1 \quad Z_{2,n+1} = \int_0^x (Z_{3,n}) dx$

$Z_{3,0} = 2 \quad Z_{3,n+1} = \int_0^x (\frac{1}{x} Z_{1,n} + Z_{3,n}) dx$

Then

$Z^P = Z_{1,0} + Z_{1,1} + Z_{1,2} + \ldots + Z_{1,P} \qquad \{3\}$

Let $Z^P = Z_{1,0} + Z_{1,1} + Z_{1,2} + \ldots + Z_{1,P}$ be the notation for an approximation to the solution with p+1 term. From {3} some computed approximation are as follows:

$Z^3 = x(1 + x + \frac{x^2}{3})$

$Z^4 = x(1 + x + \frac{x^2}{2} + \frac{x^3}{12})$

$Z^5 = x(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + + \frac{x^4}{60})$

$Z^6 = x(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{7x^4}{180} + \frac{x^5}{360})$

$Z^7 = x\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{31x^5}{4320} + \frac{x^6}{2520}\right)$

$Z^8 = x\left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{167x^6}{151200} + \frac{x^7}{20160}\right)$ ..{4}

Problem 2: Consider the delay differential equation.

$$\frac{dy}{dx} = y_0^2\left(\frac{x}{2}\right) \quad \{5\}$$

With initial condition $y(0) = 1$

Now $y_0\left(\frac{x}{2}\right) = I$

Given a linear operator

$$L = \frac{d}{dx} \quad \{6\}$$

The inverse operator is then

$$L^{-1} = \int_0^x (.)\,dx \quad \{7\}$$

Then, the equation takes the operator form,

$Ly = Ny$

Showing N as the nonlinear operator,

$Ny = y^2$

Putting L^{-1} into L in the equation. Making the L. H.S

$$L^{-1}Ly = y(x) - y(0) \quad \{8\}$$

Substituting the initial value, we have

$L^{-1}Ly = y(x) - 1$

Hence,

$$y(x) - 1 = L^{-1}(Ny) \quad \{9\}$$

Then

From the definition of the Adomian polynomials

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left(Ny(\lambda)\right)\Big|_{\lambda=0} \quad \{10\}$$

Where

The Adomian polynomial is given as

$A_0 = y_0^2$

$A_1 = 2y_0y_1$

$$A_2 = 2y_0y_2 + y_1^2 \quad \{11\}$$

Let the Recursive relation be

$$y(x) = \sin x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\{12\}$$

$y_0\left(\frac{x}{2}\right) = 1$

$y_1\left(\frac{x}{2}\right) = \frac{x}{2}$

$y_2\left(\frac{x}{2}\right) = \frac{x^2}{8}$

$y_3\left(\frac{x}{2}\right) = \frac{x^3}{48}$

$y_4\left(\frac{x}{2}\right) = \frac{x^4}{384}$

$y_5\left(\frac{x}{2}\right) = \frac{x^5}{3840}$

$y_6\left(\frac{x}{2}\right) = \frac{x^6}{46080}$

$y_7\left(\frac{x}{2}\right) = \frac{x^7}{645120}$

$y_8\left(\frac{x}{2}\right) = \frac{x^8}{10321920}$

$y_9\left(\frac{x}{2}\right) = \frac{x^9}{18579456}$

IV. DISCUSSION OF RESULTS

The recursive formula for problem 1:

From {2}

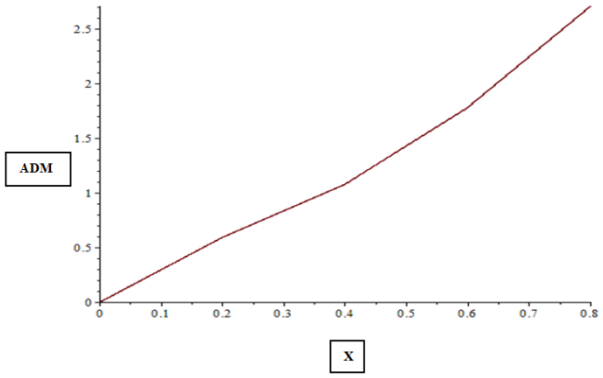
$$f_{i,n+1} = \int_0^x A_{i,n}\left(f_{i,0,\ldots,} f_{i,n}\right)dx$$

For solution of Exact we have $y(x) = x\,e^x$

X	Exact	ADM
0	0	0
0.2	0.2442805516	0.2426666667
0.4	0.5967298792	0.5941333333
0.6	1.093271280	1.073562667
0.8	1.780432742	1.777737956

Table 1

Fig 1



The general recursive formular for problem 2:

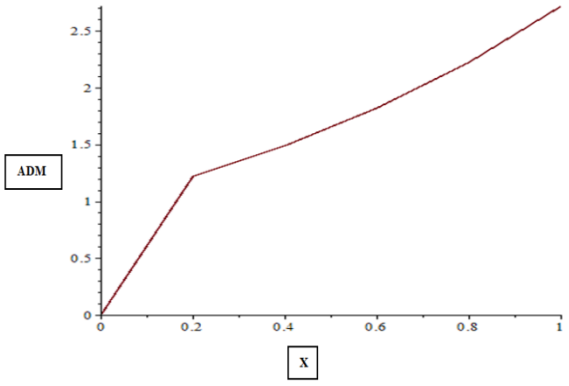
$$y(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Exact solution is given as $y(x) = \frac{1}{1-x}$

Table 2

X	Exact	ADM
0	0	0
0.2	1.300000000	1.200000000
0.4	1.700000000	1.500000000
0.6	2.490000000	1.800000000
0.8	4.900000000	2.200000000

Fig 2



V. CONCLUSION

It is demonstrated that the Adomian Decomposition Method and the Exact Method accord quite well. The graphs and tables demonstrate the graphical representation of the Adomian Decomposition Method and the Exact Method, respectively. This conclusively establishes the Adomian Decomposition Method's reliability for handling a variety of differential equation types while also providing opportunity for further research and development.

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