International Journal of Physical Science, Volume 11, Number 1, 2016
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GAP: GENERATING SET IN SEMIGROUP OF FINITE ORDER-PRESERVING FULL CONTRACTIONS

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ABSTRACT

Let $X_n = \{1, 2, \cdots, n\}$. *A (partial) transformation* $\alpha : Dom\ \alpha \subseteq X_n \to \text{Im}\ \alpha \subseteq X_n$ *is said to be full or total if* $Dom\ \alpha = X_n$, *otherwise is called strictly partial. A map* $\alpha \in Sing_n$ *is said to be order-preserving if, for all* $x, y \in X_n, x \leq y \Rightarrow x\alpha \leq y\alpha$ *and the map* $\alpha \in Sing_n$ *is said to be a contraction if, for all* $x, y \in X_n, |x\alpha \leq y\alpha| \leq |x - y|$. *In this paper, we extend the results of [3] on certain Semigroups of finite full contraction maps by computing the generating set properties of the semigroup* OCT_n, *of all order-preserving full contraction maps on the finite set* $\{1, 2, \ldots, n\}$ *through the use of Group Algorithm Programme(GAP).* ***Mathematics Subject Classification (2010). 20M20***

Keywords. *GAP, Generating Set, Contraction Map, Order-Preserving Map.*

INTRODUCTION

The important of the study of the full transformation semigroup T_X, consisting of all mappings of the set X into itself, is justified by its universal property in which every semigroup is embeddable in some T_X. There is therefore some interest in identifying and studying certain special subsemigroups of T_X. The subsemigroups of order-preserving elements of T_X have been studied in [18]. For the finite set $X_n = \{1, 2, \ldots, n\}$, the full transformation semigroup on X_n is denoted by T_n. A map α in T_n is said to be order- preserving $x \leq y \Rightarrow x\alpha \leq y\alpha$ (for all $x, y \in X_n$) and a contraction if $|x\alpha \leq y\alpha| \leq |x - y|$ (for all $x, y \in X_n$). The set of all order-preserving contraction mappings in T_n is denoted by OCT_n, and is a subsemigroup of T_n. The term contraction mapping first appeared in [17] but algebraic and combinatorial study of the semigroup OCT_n were initiated by Adeshola[1]. Many have carried out research in Partial, Full, and Partial one to one Transformation semigroup of order-preserving such as [7, 8, 9, 10, 11, 12, 13, 14, 15] but due to the fact that Contraction mappings is a new class of transformation semigroups only few researchers have worked on partial contraction mappings amongst are [23] who worked on regularity and Green's relations. Garba [11[proved the rank of the subsemigroup $\{\alpha \in PO_n : |im\,\alpha| \leq / n-2 \text{ and } \alpha \in \langle N \rangle\}$ to be equal to 6(n-2) and its nilpotent rank to be equal to 7n-15. [22] work is based on Algebraic and combinatorial study of Partial one to one contraction and Isometry and just recently [2, 3, 20] worked on ranks properties of full contraction maps. [20] initiated the algebraic study of the semigroups CI_n and OCI_n, of partial one-to-one contraction and partial one-to-one order-preserving contraction mappings of a totally ordered set $\{1, 2, \ldots, n\}$ respectively. In particular, they characterized Green's relations and their starred analogue in CI_n and OCI_n. In addition, [3, 20] investigated the ranks properties of CI_n and OCI_n. In addition, they investigated the rank properties of OCI_n.

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In their investigations, [22] listed some n elements of each of the semigroups of injective partial contraction mappings and isometries of a finite chain i.e CI_n (injective partial contraction mappings), DP_n partial isometries) and its subsemigroups consistings of ODP_n (order-preserving partial isometries), and investigated the cycle structure and the Green's relations. They were able to establish the algebraic and combinatorial properties of some of these semigroups and some results were obtained.
The methods employed in carrying out this research work were that the elements in each of the subsemigroups were listed and some tables were formed for OCT_n. The elements by their heights were arranged, within each height-class by their images sets and their kernel classes. From these tables, triangular array and sequences were formed, the patterns of the arrangement were studied and results were obtained.
The Groups, Algorithms, and Programming (GAP), in the past, the main focus of computational abstract algebra has been on group theory and commutative algebra, but due to a recent surge of interest in computing with semigroups, graphs, Lie algebras, near-rings and the like, the scope of GAP has broadened to support a more general class of mathematical structures. GAP is used both as a 'desk calculator' for automating large or tedious hand calculations, and so as a software development platform in which to implement mathematical algorithms. [21] emphasises the use of [6] as a desk calculator for semigroups.
In this study, I draw particular attention to [3] because it is just an extension of the work using Groups, Algorithms, and Programming (GAP).

Preliminaries

The set T_n, of all full transformations of X_n forms a semigroup under composition of mappings called *the full transformation semigroup*. A transformation P_n is said to be order-preserving if $(\forall x, y \in Dom\alpha)\, x \leq y \Rightarrow x\alpha \leq y\alpha$. The semigroups of order-preserving full transformation of X_n will be denoted by O_n and it was first studied by [4], and independently by [18]. Also [16] studied the relative rank of the semigroup S of all order-preserving mapping O_X on a linearly ordered set X modulo the full transformation semigroup T_X. They were able to answer the question "what is the smallest cardinality of a set A of mapping such that $\langle O_X \cup A\rangle = T_X$" that the number is one when X is countably finite or well-ordered.
For [illegible] concepts in semigroup and transformation semigroup theory, see for example [19]. Let

$$O_n = \{\alpha \in T_n : (\forall x, y \in X_n)\, x \leq y \Rightarrow x\alpha \leq y\alpha \text{ or } x\alpha \geq y\alpha\} \qquad (1)$$

be the subsemigroup of T_n consisting of all order-preserving full transformations of X_n. Also let

$$CT_n = \{\alpha \in T_n : (\forall x, y \in X_n)\, |x - y| \geq |x\alpha - y\alpha|\} \qquad (2)$$

be the subsemigroup of T_n consisting of all *full contractions* of X_n.
Let O_n and CT_n be as defined in equations (1) and (2) respectively and let

$$OCT_n = O_n \cap CT_n. \qquad (3)$$

Let

$$\alpha = \begin{pmatrix} A_1 & A_2 & \cdots & A_p \\ a_1 & a_2 & \cdots & a_p \end{pmatrix},$$

where $a_1 < a_2 < \cdots < a_p$, $A_1 < A_2 < \cdots < A_p$ and the partition $\{A_1, A_2, \cdots, A_p\}$ will be called convex partition see [1, 2, 3, 20] for details. Some possible image sets for elements in K_{n-1} are as follows

$$\delta_{i,i+1}=\begin{pmatrix}1\ 2\cdots i-1\ \{i,i+1\}\ i+2\cdots\ n\\ 1\ 2\cdots i-1\quad i\quad i+1\cdots n-1\end{pmatrix}\qquad (2\le i\le n-2),$$

$$\delta'_{i,i+1}=\begin{pmatrix}n\ n-1\cdots i+2\ \{i,i+1\}\ i-1\cdots\ 1\\ 1\quad 2\quad\cdots i-1\quad i\quad i+1\cdots n-1\end{pmatrix}\qquad (2\le i\le n-2),$$

$$\gamma_{i,i+1}=\begin{pmatrix}1\quad 2\quad\cdots i-1\ \{i,i+1\}\ i+2\cdots\ n\\ n-1\ n-2\cdots i+1\quad i\quad i-1\cdots\ 1\end{pmatrix}\qquad (2\le i\le n-2),$$

$$\gamma'_{i,i+1}=\begin{pmatrix}n\quad n-1\quad\cdots i+2\ \{i,i+1\}\ i-1\cdots\ 1\\ n-1\ n-2\quad\cdots i+1\quad i\quad i-1\cdots\ 1\end{pmatrix}\qquad (2\le i\le n-2).$$

From the array of table mentioned in the introduction we have

$$A=\{\delta_1,\delta_2\ldots..\delta_{n-1}\}$$

and

$$K_{n-1}=\{\lambda_1,\ldots,\ \lambda_{n-1},\gamma_1,\ldots..\gamma_{n-1}\}.$$

Semigroups of Order-Preserving Contraction

Each map $\alpha\in O_n$ can be written as

$$\alpha=\begin{pmatrix}A_1\ A_2\cdots A_p\\ a_1\ a_2\cdots a_p\end{pmatrix},\qquad (4)$$

where $im(\alpha)=\{a_1,a_2,\ldots,a_p\}$ and $A_1,A_2,\ldots,A_p$ are equivalence classes under the equivalence $\ker\alpha=\{(x,y)\in X_n\times X_n : x\alpha=y\alpha\}$. Thus, $x\alpha=a_i$ whenever $x\in A_i\ (1\le i\le p)$.

Lemma 3.1 *[3, Lemma 5.5]* $rank\ (OCT_n)\le n-1$.

Proof:

As defined in equations (3) and (4), it follows that any generating set for K_{n-1} $(or\ OCT_n)$ must pick from each of the possible Kernel classes and also from each of the possible image sets in K_{n-1}. Thus, $rank\ (OCT_n)\ge n-1$.

We give examples to illustrate:

Example 1: For elements $A=\{\delta_{1,2},\delta_{2,3}\}$ in OCT_3 with the same rank, $rank\ (OCT_3)=n-1$.

$$\delta_{1,2}=\begin{pmatrix}\{1,2\}\ \{3\}\\ 1\quad 2\end{pmatrix},\qquad \delta_{2,3}=\begin{pmatrix}\{1\}\ \{2,3\}\\ 2\quad 3\end{pmatrix}$$

Example 2: For elements $A = \{\delta_{1,2}, \delta_{2,3}, \delta_{3,4}\}$ in OCT_4 with the same rank, $rank\ (OCT_4) = n-1$.

$$\delta_{1,2} = \begin{pmatrix} \{1,2\} & \{3\} & \{4\} \\ 1 & 2 & 3 \end{pmatrix}, \qquad \delta_{1,3} = \begin{pmatrix} \{2\} & \{1,3\} & \{4\} \\ 1 & 2 & 3 \end{pmatrix}, \qquad \delta_{3,4} = \begin{pmatrix} \{1\} & \{2\} & \{3,4\} \\ 2 & 3 & 4 \end{pmatrix}.$$

Example 2: For elements $A = \{\delta_{1,2}, \delta_{2,3}, \delta_{3,4}, \delta_{4,5}\}$ in OCT_5 with the same rank, $rank\ (OCT_5) = n-1$.

$$\delta_{1,2} = \begin{pmatrix} \{1,2\} & \{3\} & \{4\} & \{5\} \\ 1 & 2 & 3 & 4 \end{pmatrix}, \qquad \delta_{2,3} = \begin{pmatrix} \{1\} & \{2,3\} & \{4\} & \{5\} \\ 1 & 2 & 3 & 4 \end{pmatrix},$$

$$\delta_{3,4} = \begin{pmatrix} \{1\} & \{2\} & \{3,4\} & \{5\} \\ 1 & 2 & 3 & 4 \end{pmatrix}, \qquad \delta_{4,5} = \begin{pmatrix} \{1\} & \{2\} & \{3\} & \{4,5\} \\ 2 & 3 & 4 & 5 \end{pmatrix}.$$

Remark: Note that the semigroup OCT_n is certainly not regular. For example, if $n \geq 4$,

(1).

$$\alpha = \begin{pmatrix} \{1\} & \{2,3\} & \{4\} \\ 1 & 2 & 3 \end{pmatrix} \in OCT_4.$$

Then by [1, 20, 23], α is not Regular in OCT_4.

(2).

$$\alpha = \begin{pmatrix} \{1\} & \{2\} & \{3, 4\} & \{5\} \\ 2 & 3 & 4 & 5 \end{pmatrix} \in OCT_5.$$

Then by [1, 20, 23], α is not Regular in OCT_5.

Lemma 3.2 For any $n = 3$, $rank(OCT_n) = n-1$.

Proof:

```
gap> a := Transformation([2, 3, 3]);
Transformation([2, 3, 3])
gap> b := Transformation([1, 1, 2]);
Transformation([1, 1, 2])
gap> s := Semigroup(a, b);
<semigroup with 2 generators>
gap> Size(s);
```

$7 \to \begin{pmatrix} 1 & 2 \cdots n \\ 1 & 2 \cdots n \end{pmatrix}$, (the identity element).

gap> Elements(s);
Transformation([1, 1, 1]), Transformation([1, 1, 2]),
Transformation([1, 2, 2]), Transformation([2, 2, 2]),
Transformation([2, 2, 3]), Transformation([2, 3, 3]),
Transformation([3, 3, 3]).

Lemma 3.3 For any $n = 4$, $rank(OCT_n) = n-1$.

Proof:
gap> a := Transformation ([2, 3, 4, 4]);
Transformation([2, 3, 4, 4])
gap> b := Transformation ([1, 2, 2, 3]);
Transformation ([1, 2, 2, 3])
gap> c := Transformation ([1, 1, 2, 3]);
Transformation ([1, 1, 2, 3])
gap> s := Semigroup (a, b, c);
< semigroup with 3 generators >
gap> Size(s);

$19 \to \begin{pmatrix} 1 & 2 \cdots n \\ 1 & 2 \cdots n \end{pmatrix}$, (the identity element).

gap> Elements(s);
[Transformation([1, 1, 1, 1]), Transformation([1, 1, 1, 2]),
Transformation([1, 1, 2, 2]), Transformation([1, 1, 2, 3]),
Transformation([1, 2, 2, 2]), Transformation([1, 2, 2, 3]),
Transformation([1, 2, 3, 3]), Transformation([2, 2, 2, 2]),
Transformation([2, 2, 2, 3]), Transformation([2, 2, 3, 3]),
Transformation([2, 2, 3, 4]), Transformation([2, 3, 3, 3]),
Transformation([2, 3, 3, 4]), Transformation([2, 3, 4, 4]),
[illegible], Transformation([3, 3, 3, 3]), Transformation([3, 3, 3, 4]),
Transformation([3, 3, 4, 4]), Transformation([3, 4, 4, 4]),
Transformation([4, 4, 4, 4])]

Lemma 3.4 For any $n = 5$, $rank(OCT_n) = n-1$.

Proof:
gap> a := Transformation ([2, 3, 4, 5, 5]);
Transformation([2, 3, 4, 5, 5])
gap> b := Transformation ([1, 2, 3, 3, 4]);
Transformation ([1, 2, 3, 3, 4])
gap> c := Transformation ([1, 2, 2, 3, 4]);
Transformation ([1, 2, 2, 3, 4])
gap> d := Transformation ([1, 1, 2, 3, 4]);
Transformation ([1, 1, 2, 3, 4])
gap> s := Semigroup (a, b, c, d);
< semigroup with 4 generators >
gap> Size(s);

$47 \to \begin{pmatrix} 1 & 2 \cdots n \\ 1 & 2 \cdots n \end{pmatrix}$, (the identity element).

gap> Elements(s);

[Transformation([1, 1, 1, 1, 1]), Transformation([1, 1, 1, 1, 2]),
Transformation([1, 1, 1, 2, 2]), Transformation([1, 1, 1, 2, 3]),
Transformation([1, 1, 2, 2, 2]), Transformation([1, 1, 2, 2, 3]),
Transformation([1, 1, 2, 3, 3]), Transformation([1, 1, 2, 3, 4]),
Transformation([1, 2, 2, 2, 2]), Transformation([1, 2, 2, 2, 3]),
Transformation([1, 2, 2, 3, 3]), Transformation([1, 2, 2, 3, 4]),
Transformation([1, 2, 3, 3, 3]), Transformation([1, 2, 3, 3, 4]),
Transformation([1, 2, 3, 4, 4]), Transformation([2, 2, 2, 2, 2]),
Transformation([2, 2, 2, 2, 3]), Transformation([2, 2, 2, 3, 3]),
Transformation([2, 2, 2, 3, 4]), Transformation([2, 2, 3, 3, 3]),
Transformation([2, 2, 3, 3, 4]), Transformation([2, 2, 3, 4, 4]),
Transformation([2, 2, 3, 4, 5]), Transformation([2, 3, 3, 3, 3]),
Transformation([2, 3, 3, 3, 4]), Transformation([2, 3, 3, 4, 4]),
Transformation([2, 3, 3, 4, 5]), Transformation([2, 3, 4, 4, 4]),
Transformation([2, 3, 4, 4, 5]), Transformation([2, 3, 4, 5, 5]),
Transformation([3, 3, 3, 3, 3]), Transformation([3, 3, 3, 3, 4]),
Transformation([3, 3, 3, 4, 4]), Transformation([3, 3, 3, 4, 5]),
Transformation([3, 3, 4, 4, 4]), Transformation([3, 3, 4, 4, 5]),
Transformation([3, 3, 4, 5, 5]), Transformation([3, 4, 4, 4, 4]),
Transformation([3, 4, 4, 4, 5]), Transformation([3, 4, 4, 5, 5]),
Transformation([3, 4, 5, 5, 5]), Transformation([4, 4, 4, 4, 4]),
Transformation([4, 4, 4, 4, 5]), Transformation([4, 4, 4, 5, 5]),
Transformation([4, 4, 5, 5, 5]), Transformation([4, 5, 5, 5, 5]),
Transformation([5, 5, 5, 5, 5])].

Lemma 3.5 For any $n = 6$, $rank(OCT_n) = n-1$.

Proof:

gap > a := Transformation ([2, 3, 4, 5, 6, 6]);
Transformation([2, 3, 4, 5, 6, 6])
gap > b := Transformation ([1, 2, 3, 4, 4, 5]);
Transformation ([1, 2, 3, 4, 4, 5])
gap > c := Transformation ([1, 2, 3, 3, 4, 5]);
Transformation ([1, 2, 3, 3, 4, 5])
gap > d := Transformation ([1, 2, 2, 3, 4, 5]);
Transformation ([1, 2, 2, 3, 4, 5])
gap > d := Transformation ([1, 1, 2, 3, 4, 5]);
Transformation ([1, 1, 2, 3, 4, 5])
gap > s := Semigroup (a, b, c, d, e);
semigroup with 5 generators >
gap > Size(s);

111 + $\begin{pmatrix} 1 & 2 \cdots n \\ 1 & 2 \cdots n \end{pmatrix}$, (the identity element).

gap> Elements(s);
[Transformation([1, 1, 1, 1, 1, 1]), Transformation([1, 1, 1, 1, 1, 2]),
Transformation([1, 1, 1, 1, 2, 2]), Transformation([1, 1, 1, 1, 2, 3]),
Transformation([1, 1, 1, 2, 2, 2]), Transformation([1, 1, 1, 2, 2, 3]),
Transformation([1, 1, 1, 2, 3, 3]), Transformation([1, 1, 1, 2, 3, 4]),
Transformation([1, 1, 2, 2, 2, 2]), Transformation([1, 1, 2, 2, 2, 3]),
Transformation([1, 1, 2, 2, 3, 3]), Transformation([1, 1, 2, 2, 3, 4]),
Transformation([1, 1, 2, 3, 3, 3]), Transformation([1, 1, 2, 3, 3, 4]),
Transformation([1, 1, 2, 3, 4, 4]), Transformation([1, 1, 2, 3, 4, 5]),..

Transformation([1, 2, 2, 2, 2, 2]), Transformation([1, 2, 2, 2, 2, 3]),
Transformation([1, 2, 2, 2, 3, 3]), Transformation([1, 2, 2, 2, 3, 4]),
Transformation([1, 2, 2, 3, 3, 3]), Transformation([1, 2, 2, 3, 3, 4]),
Transformation([1, 2, 2, 3, 4, 4]), Transformation([1, 2, 2, 3, 4, 5]),
Transformation([1, 2, 3, 3, 3, 3]), Transformation([1, 2, 3, 3, 3, 4]),
Transformation([1, 2, 3, 3, 4, 4]), Transformation([1, 2, 3, 3, 4, 5]),
Transformation([1, 2, 3, 4, 4, 4]), Transformation([1, 2, 3, 4, 4, 5]),
Transformation([1, 2, 3, 4, 5, 5]), Transformation([2, 2, 2, 2, 2, 2]),
Transformation([2, 2, 2, 2, 2, 3]), Transformation([2, 2, 2, 2, 3, 3]),
Transformation([2, 2, 2, 2, 3, 4]), Transformation([2, 2, 2, 3, 3, 3]),
Transformation([2, 2, 2, 3, 3, 4]), Transformation([2, 2, 2, 3, 4, 4]),
Transformation([2, 2, 2, 3, 4, 5]), Transformation([2, 2, 3, 3, 3, 3]),
Transformation([2, 2, 3, 3, 3, 4]), Transformation([2, 2, 3, 3, 4, 4]),
Transformation([2, 2, 3, 3, 4, 5]), Transformation([2, 2, 3, 4, 4, 4]),
Transformation([2, 2, 3, 4, 4, 5]), Transformation([2, 2, 3, 4, 5, 5]),
Transformation([2, 2, 3, 4, 5, 6]), Transformation([2, 3, 3, 3, 3, 3]),
Transformation([2, 3, 3, 3, 3, 4]), Transformation([2, 3, 3, 3, 4, 4]),
Transformation([2, 3, 3, 3, 4, 5]), Transformation([2, 3, 3, 4, 4, 4]),
Transformation([2, 3, 3, 4, 4, 5]), Transformation([2, 3, 3, 4, 5, 5]),
Transformation([2, 3, 3, 4, 5, 6]), Transformation([2, 3, 4, 4, 4, 4]),
Transformation([2, 3, 4, 4, 4, 5]), Transformation([2, 3, 4, 4, 5, 5]),
Transformation([2, 3, 4, 4, 5, 6]), Transformation([2, 3, 4, 5, 5, 5]),
Transformation([2, 3, 4, 5, 5, 6]), Transformation([2, 3, 4, 5, 6, 6]),
Transformation([3, 3, 3, 3, 3, 3]), Transformation([3, 3, 3, 3, 3, 4]),
Transformation([3, 3, 3, 3, 4, 4]), Transformation([3, 3, 3, 3, 4, 5]),
Transformation([3, 3, 3, 4, 4, 4]), Transformation([3, 3, 3, 4, 4, 5]),
Transformation([3, 3, 3, 4, 5, 5]), Transformation([3, 3, 3, 4, 5, 6]),
Transformation([3, 3, 4, 4, 4, 4]), Transformation([3, 3, 4, 4, 4, 5]),
Transformation([3, 3, 4, 4, 5, 5]), Transformation([3, 3, 4, 4, 5, 6]),
Transformation([3, 3, 4, 5, 5, 5]), Transformation([3, 3, 4, 5, 5, 6]),
Transformation([3, 3, 4, 5, 6, 6]), Transformation([3, 4, 4, 4, 4, 4]),
Transformation([3, 4, 4, 4, 4, 5]), Transformation([3, 4, 4, 4, 5, 5]),
Transformation([3, 4, 4, 4, 5, 6]), Transformation([3, 4, 4, 5, 5, 5]),
Transformation([3, 4, 4, 5, 5, 6]), Transformation([3, 4, 4, 5, 6, 6]),
Transformation([3, 4, 5, 5, 5, 5]), Transformation([3, 4, 5, 5, 5, 6]),
Transformation([3, 4, 5, 5, 6, 6]), Transformation([3, 4, 5, 6, 6, 6]),
Transformation([4, 4, 4, 4, 4, 4]), Transformation([4, 4, 4, 4, 4, 5]),
Transformation([4, 4, 4, 4, 5, 5]), Transformation([4, 4, 4, 4, 5, 6]),
Transformation([4, 4, 4, 5, 5, 5]), Transformation([4, 4, 4, 5, 5, 6]),
Transformation([4, 4, 4, 5, 6, 6]), Transformation([4, 4, 5, 5, 5, 5]),
Transformation([4, 4, 5, 5, 5, 6]), Transformation([4, 4, 5, 5, 6, 6]),
Transformation([4, 4, 5, 6, 6, 6]), Transformation([4, 5, 5, 5, 5, 5]),
Transformation([4, 5, 5, 5, 5, 6]), Transformation([4, 5, 5, 5, 6, 6]),
Transformation([4, 5, 5, 6, 6, 6]), Transformation([4, 5, 6, 6, 6, 6]),
Transformation([5, 5, 5, 5, 5, 5]), Transformation([5, 5, 5, 5, 5, 6]),
Transformation([5, 5, 5, 5, 6, 6]), Transformation([5, 5, 5, 6, 6, 6]),
Transformation([5, 5, 6, 6, 6, 6]), Transformation([5, 6, 6, 6, 6, 6]),
Transformation([6, 6, 6, 6, 6, 6])]

Theorem 3.6 *[3, Theorem 5.6]* For any $n \geq 2$, $rank(OCT_n) = n-1$.

CONCLUSION

This paper is strongly an extension of work and results in [3]. We only present those results of $|OCT_n|$ using Groups, Algorithms, and Programming(GAP). The summary of the number of elements generated is

as shown on *Table* 4.1 below. Investigation using Groups, Algorithms, and Programming(GAP) can also be looked into in Semigroup of Order-Decreasing Contraction, DCT_n, Semigroup of Order-Preserving o Order-Reversing Contraction, $ORCT_n$, Semigroup of Preserving or Order-Decreasing Contraction $ODCT_n$.

n/k	1	2	3	4	5	6	$\|OCT_n\|$
1	1						1
2	2	1					3
3	3	4	1				8
4	4	9	6	1			20
5	5	16	18	8	1		48
6	6	25	40	30	10	1	112

Table 4.1.

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BLA-2016-088/ I. S. Udofia/blackwelljournal@yahoo.com/01-03-2016