SEMI-GROUPS IN AUTOMATA

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ABSTRACT

There are three types of theoretical machines namely Finite Automata, Pushdown Automata and Turning Machines. Each of these machines has different capability to recognize symbols and patterns. In this work, we looked at the simplest of these three machines, that is, the Finite Automata and we drew a relationship between it an the theory of Semigroup, Free Semigroups in particular.

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INTRODUCTION

A binary operation from $X \times Y$ to Z is a mapping from $X \times Y$ to Z A binary operation.
$*: X \times Y \rightarrow X$ is called a binary operation on X, if $*: X \times Y \rightarrow Z$ is a binary operation usually written as:
$a * b$ for $*(a,b)$ then a binary operation $*$ on a set S is associative if

$(a*b)*c = a*(b*c)$ for all $a,b,c, \in S$ is a Semi-group [4,7, 8,].

In [6, 9] an automation is a mathematical study of machines and their capabilities for solving problem by means of Algorithms. Also in [2,5] automation is a mathematical model for a finite state machine(*FSM*). An *(FSM)* is a machine that given an input of symbols, jumps through a series of state according to a transitive function (which can be expressed as a table). This transition tells the automation which state go to next given a current state and a current symbol.

PRELIMINARIES

Finite Automata: In theoretical computer science, automata theory is the study of abstract machine and problems they are able to solve.

In [5], let S be a finite set and let Σ be a finite set of symbols. Also, let δ be a function from $X \times \Sigma$ to S, let q_0 be a state in S and let A be a subset of S. We call the elements of S a state, δ the transition function, q_0 the initial state and A the set of accepting states. Then a finite automation is a 5 - tuple $\{S, \Sigma, q_0, \delta, A\}$.

Consider the transition diagram below where finite automation is denoted $M = \{S, \Sigma, q_0, \delta, A\}$,
$S = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $A = \{q_0\}$ and δ is shown in the table below

States	Input	
	0	1
q_0	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

Suppose 110101 is input to M. We note that $\delta(q_o, 1) = q_1$ *and* $\delta(q_1, 1) = q_o$
Thus, $\delta(q_o, 11) = \delta(\delta(q_o, 1), 1) = \delta(q_1, 1) = q_o$

We might remark that this $L(M)$, that is, Language Accepted by M, but we are interested in110101.We continue by noting that $\delta(q_o, 0) = q_2$. Thus, $\delta(q_o, 110) = \delta(\delta(q_o, 11), 0) = \delta(q_o, 0) = q_2$ continue in this fashion, we find that, $\delta(q_o, 11) = q_3$, $\delta(q_o, 11010) = q_1$ and finally $\delta(q_o, 110101) = q_o$. The entire sequence of states is

1 1 0 1 0 1

$q_o\ q_1\ q_o\ q_2\ q_3\ q_1\ q_o$

Thus 110101 is in $L(M)$. This is a set of strings with an even number of 0's and even number of 1's.

Free Semi-groups: In abstract algebra, the free moniod on a set A is the monoid whose elements are all the finite sequence (or string) of zero or more elements from A with the binary operation of Concatenation. It is usually denoted A^* while Free Semi-groups is denoted A^*.

From [7] and [8], let A be a non-empty set and let A^* be the set of all finite, non-empty words $a_1a_2....a_x$ in the alphabet A. By concatenation, a binary operation is defined on A^* as

$(a_1 a_2.....a_x)(b_1 b_2....b_y) = a_1a_2.....a_xb_1b_2...b_y$. Then, with respect to this operation, A^* is a Free Semi-group on A while the set A is a generating set for A.

SOME PROPERTIES OF FREE SEMI-GROUPS

Proposition 2.1: The free monoid A^* is cancellative. A Semi-group or (moniod) S is called equidivisible if, for all s,t,u,v *in* S,

$st = vu$ implies either

(i) there exists $x = S^1$ such that
$S = ux$ *and* $v = xt$ *or*

(ii) there exists y *in* S^1 such that $u = sy$ *and* $t = yv$
From $u^{-1}S$ *and* $y = S^{-1}u$ respectively.

Proposition 2.2: The free monoid A* is equidivisible

Proof

Let $st = uv = a_1a_2.....a_m$

Where $a_1a_2...a_m \in A$

Then,
for some k, 1 *in* $\{0, 1, \ldots, m\}$

$S = a_1 a_2 \ldots\ldots a_k$, $t = a_{k+1}\ a_{k+2} \ldots\ldots a_m$

$S = a_1 a_2 \ldots\ldots a_t$, $t = a_{t+1}\ a_{t+2} \ldots\ldots a_m$

if

$k \geq 1$, *then* $x = a_{t+1} \ldots\ldots a_k$,

If $k \leq 1$, *then* $y = a_{k+1} \ldots\ldots a_1$, for (i) and (ii) respectively.

Theorem 2.1

Let A be a non-empty set and let S be a Semi-group if $\Phi : A \to S$ in an arbitrary mapping then there exist a unique homomorphism $\varphi : A^+ \to S$ such that $\varphi / A = \Phi$

Proof

Define $\varphi : A^+ \to S$ by

$$(a_1\ a_2 \ldots\ldots a_m)\varphi = (a_1\ \Phi)(a_2\ \Phi)\ldots\ldots(a_m\ \Phi)$$

Then,
it is evident that φ is a homomorphism and that $\varphi / A = \Phi$ Moreover φ is unique for if $\varphi : A^+ \to S$ is to have the required properties
Then,

$$(a_1\ a_2 \ldots\ldots a_m)\varphi = (a_1\ \Phi)(a_2\ \Phi)\ldots\ldots(a_m\ \Phi)$$
$$(a_1\ \Phi)(a_2\ \Phi)\ldots\ldots(a_m\ \Phi)$$
$$(a_1\ a_2 \ldots\ldots a_m)\varphi$$

for every $a_1 a_2 \ldots\ldots a_m$ *in* A^+, *and so* $\varphi = \varphi$

CONNECTION BETWEEN FREE SEMI-GROUPS AND AUTOMATA

An alphabet Σ is a finite, non-empty set of elements. The elements of the alphabet are usually called symbols or letters. Let

$D = \{0, 1, 2, 3, \ldots\ldots\ldots\ldots, 9\}$ and

$A = \{a, b, c, d, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, z\}$ a word over an alphabet Σ is a finite sequence of symbols from Σ, usually written without any separating commas.

For example

25801
45639

are words over D, while

blackdog

are words over A.

Similarly, given an alphabet $\{a,b,c\}$, the words

Σ *cab abb*

are words over $\{\{a,b,c\}$, while the concatenation of *street* with *boy* is

streetboy.

Strings are usually made up of symbols available on a computer keyboard through binary numbers 0*s* and 1*s*. These binary numbers are later processed through method of pattern recognition problems learnt in Finite Automata Machines. Finite Automata Machines are thought of as processing these strings of letters over some alphabet. Since the elements of Free Semi-groups and Free monoids are words in alphabet then they are strings of symbols in which we recognized that the strings that must be recognized in any application are constructed from set of symbols. These elements are strings of symbols and every input is an element of A^+, where A is the set of space and punctuation symbols together with both upper and lower case letters of the alphabet. Therefore, automata machines can be seeing as we see Free Semi-groups in the generating set A where $A = \{0, 1\}$, so the principles of Semi-groups such as congruence and idempotent can be made use of in Automata.

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