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Sum-Difference Model for Production of a Star-Like Harmonic Oscillator Transformation Semigroup

Akinwunmi, S. A.[1], Bakare, G.N.[2], Ogbu, A.O.[3], Garba, R.I[4]
[1]Department of Mathematics & Statistics, Federal University of Kashere, Gombe, [2]Department of Mathematics, University of Ilorin, [3]Department of Mathematics, Federal College of Education (Tech) Bichi, Kano,& [4]Department of Statistics and Mathematical Science, Kwara State University Malete
Email: [1]sakinwunmi@fukashere.edu.ng, [2]bakare.gn@unilorin.edu.ng, [3]angel6tus@gmail.com, [4]risqot.ibrahim@kwasu.edu.ng

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Abstract

This study introduces a novel sum-difference model for generating a star-like harmonic oscillator transformation semigroup, based on combinatorial properties of partial transformations over the finite set $Z_n = \{1, 2, 3\ldots, n\}$. *A partial transformation on* Z_n *is defined as a mapping from a subset* α^*. *Using this framework, we construct a generalized two-dimensional star-like harmonic structure to produce ordered sequences and visualizable transformations. Specifically, for transformations* $\alpha^*, \beta^*, \gamma^* \in H\omega_n^*$, *the cardinality* $|H\omega_n^*|$ *is modeled as* $\frac{1}{2}V^{\gamma^*} = \frac{G^{\alpha^*} - H^{\beta^*}}{2} 4\alpha^*\beta^*$ *which governs the generation of star-like sequences and image mappings. The paper presents a unified analytical approach to deriving these results using a star-like sum-difference operator. We further establish new harmonic oscillator relations within the semigroup, supported by proofs of key combinatorial functions. This research contributes to the development of algebraic methods in harmonic transformation theory and lays a foundation for future applications in signal modeling, automata theory, and abstract algebraic systems.*

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1. Introduction

The theory of algebraic semigroups, or monoids, emerged during a time in mathematics when group and ring theories were well-established sciences, influencing not just algebra as a whole but also its scientific applications.
Prominent mathematicians such as [1], [2], [3], [4], [5], [6], and recently [7] have made a number of fascinating discoveries while researching the algebraic and combinatorial properties of transformation semigroups. Their research has resulted in practical instruments that are employed in a wide range of combinatorial mathematics applications. However, equivalent results for the generation of star-like harmonic oscillators have not kept pace with all of these discoveries, making this model important. Furthermore, the numerical application of this transformation semigroup to theoretical physics needs to be determined.
The current research focuses on finite sets and their transformations. Let Z be a finite set, say $Z = \{z_1, z_2, z_3 \ldots z_n\}$, where n is a nonnegative integer. The transformation of Z is an array of the following form:

$$\alpha^* = \begin{pmatrix} z_1 & z_2 & z_n \\ y_1 & y_2 & y_n \end{pmatrix}, \qquad (1)$$

where all $y_i \in Z$. The element y_i will be called the value of the transformation α^* at the element k and will be denoted by $\alpha^*(y)$. The fact that α^* is a transformation of Z is usually written as $\alpha^* = Z \to Z$ As the nature of elements of Z is not important for us, instead of Z we shall usually consider the set $Z = Z_n = \{1,2, \ldots n\}$. The partial star-like transformations of Z, that is, transformations of the form $\alpha^* = A \to Z$, where $A = \{l_1, l_2, \ldots . l_k\}$. is a subset of Z is
given as

$$\alpha^* = \begin{pmatrix} l_1 & l_2 \ldots . & l_k \\ \alpha^*(l_1) & (l_2) \ldots . & \alpha^*(l_k) \end{pmatrix}. \qquad (2)$$

We may also write $\alpha^* = Z \to Z$ for a partial star-like transformation, having in mind that such α^* is only defined on some elements from Z. [3]. A star-like transformation semigroup is said to satisfy collapse function if and only if it is not already monoid and if the collapse $C^+(\alpha^*) = \left|\cup t\alpha^{*-1} : t \in Pt_{\alpha^*}\omega_n^*\right| \; and \; \left|t\alpha^{*-1}\right| \geq 2$ [8]. [1] noted that a semigroup is a non-empty set which is closed under an associative binary operation. Using the sum-difference linear system, this paper establishes a model suitable for processing oscillatory sequences and establishes efficient oscillatory sequences and establishes efficient oscillatory mitigation process.

There are many examples of different classes of semigroups, but the classical ones are obtained by mapping of a set into itself. This is because self of a set play similar role in semigroup theory as permutations in the theory of groups. That is, every semigroup can be represented by a semigroup of mapping of a set.

The study of various finite semigroups of transformations contributes significantly to semigroup theory in the same way that the study of finite symmetric and alternating groups does; thus, several notable discoveries have resulted from the research of some well-known mathematicians [9], [10], [11], [12], and [13].

In actuality, the study of semigroups of full T_n, partial P_n, and partial one-one has been advantageous over time. It is worth noting that this research was motivated by the findings of [4], [8], and [14]. The paper provides useful insights for future researchers by developing a model based on physical science. Based on the research of [15], we discovered that the theory of star-like harmonic oscillAators can be applied in almost every scientific, technological, engineering, and mathematical sector due to its broad range of physical science representations.

2. Basic Preliminary

Let $H\omega_n^*$ *be the star-like harmonic oscillator transformation, then by the principle of* $\emptyset$ empty map, a star-like harmonic mapping $\alpha^*: M \to M$ is a rule that take from a sub-set $N = D(\alpha^*)$ of M *onto* $I(\alpha^*) = \alpha^*(N) \subseteq M$ such that $M = \{1....5\}$ and $D(\alpha^*) = \{2,4,5\}$ with $\alpha^*(2) = 1, \alpha^*(4) = 5$ and $\alpha^*(5) = 1$.

Suppose that α^*, β^* are starlike transformations and $f^*: \alpha^* \to \beta^*$ is a star-like rule. Then f^* is Star-like composite invertible if there exists a star-like rule $g^*: \beta^* \to \alpha^*$ such that $g^* \circ f^* = e^*_{\alpha^* i} \; and \; f^* \circ g^* = e^*_{\beta^* i}$[8]. Let X and Y be the two non empety sets such that there is some rule F which assigns to each element $y \in Y$, a unique element $x \in X$, then this rule is said to be a transformation or mapping. [14]. For any real number and all natural numbers b and c we have $\sum_{j=0}^{i} (a - j)\begin{bmatrix} b+j \\ j \end{bmatrix} = (a - c - 1)\begin{bmatrix} b+c+1 \\ c \end{bmatrix} + \begin{bmatrix} b+c+2 \\ c \end{bmatrix}$ [2]. We discovered from the study of the existing that some harmonic oscillator can be established and standardize using the star-like harmonic oscillator model to produce the star-like image and sequence.

Definition 1: Star-like Folding principle: Star-like folding principle: this involves a process in which a transparent A4 sheet is folded in star-like form and then some star-like sequence and prisms were represented.

Definition2: Star-Like Harmonic Oscillator Semigroup: The rate and frequency occurrence of the composition of the star-like operator on the two-dimension transformation to image and sequence production.

Definition 3: Star-like Faces H^* : This is a star-like flat or curving surface of a star-like transformation semigroup.

Definition 4: Star-like Edges G^* : This is a star-like disk point e^{u_0} where two or more star-like faces meet.

Definition 5: Star-like Vertices V^* : This is a star-like transformation corner where two or more star-like edges meet.

3. The Production of Star-Like Harmonic Oscillator Transformation Semigroup

Any extended star-like transformation $\alpha^*_{i,j} \in H\omega^*_n$ with a star-like disk point e^{u_0} is a star-like harmonic oscillator if it adheres to the star-like operator in definition (1) and star-like folding principle. This classification is assigned when the faces H^*, edges G^*, and vertices V^* converge at a star-like disk point $e^{u_0} = |\wr^*|$ with a 360-degree angular measure, and the transformation satisfies both combinatorial and the oscillatory model using ∂_h to represent the star-like oscillatory points. We consider a star-like set and a star-like finite transformation semigroup as the foundation for our study, which builds on previous studies, providing novel insights and advancing the current understanding of semigroups. Furthermore, the methodology used in this paper ensures robustness and reliability, enhancing the credibility of our findings. We demonstrated the physical building of the new semigroup using A4 plain paper, inferring star-like relations using a sum-difference model that crosses the gap between geometry and physics research. The study's findings have the potential to inform future research and increase knowledge in algebra and geometrical physics.

The star-like harmonic oscillator model presented in Figures 1 and 2 is based on the harmonic oscillator of the star-like two-dimensional transformation of Z_n.

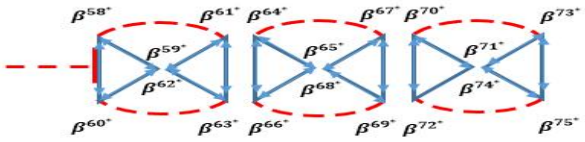


Figure 1: **Star-like Harmonic oscillator model**

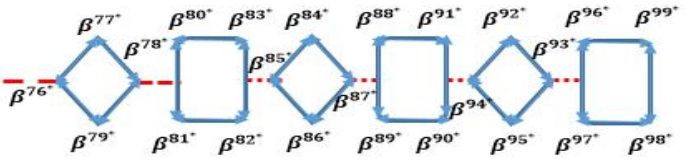


Figure 2: **Star-like image and sequence production model**

Then, adopting the star-like operator of [8], we obtain the harmonic oscillator array of Z_n such that $\gamma^* H\omega_n^*$ is a star-like model for all two-dimensional transformation harmonic oscillator characterization to image and sequence production shown in Equation (3) below

$$H\omega_n^* = \begin{pmatrix} z_n & y_n & z_{n+1} & y_{n+1}\cdots & zy_n \\ \gamma^* Hz_n & \gamma^* Hy_{n+1} & \gamma^* Hz_{n+1} & \gamma^* Hy_{n+1}\cdots & \gamma^* Hz_n y_n \end{pmatrix}. \quad (3)$$

Where all $HZ_n y_n \in H\omega_n^*$ with the core value of the star-like transformation determined by its harmonic oscillator order in relation to the recursive relation for each order of set Z_n. This section describes the fundamental methodology for investigating the translation of star-like two-dimensional harmonic oscillators into image and sequence generation. The partial transformation semigroup's elements were obtained using the general formula:

$$|P_n| = (n+1)^n, \quad \forall\, n \geq 1. \quad (4)$$

Thus, we use different cases of combinatorial principles on Equation (3) to establish the harmonic oscillatory order of any star-like transformation from the models in Figures (1) and (2) that lead to the generation of the star-like image and sequence. The approach used to generate the star-like oscillatory sequences that serve as the basis for the primary results is as follows:

(i) Decompose any star-like transformation $\alpha^*_{i,j} \in H\omega^*_n$ to a linear or quadratic algebraic equation such that $\alpha^*_{i,j} + \alpha *^2_{i,j} + \alpha *^3_{i,j} + \ldots + \alpha *^n_{i,j} = \Delta_n(z)$.

(ii) Obtain the oscillatory values of $\alpha^*_{i,j}$.

(iii) Factorize the values obtained in (ii) to obtain the star-like oscillator recursive order of $H\omega^*_n$.

4. Main Result

The research's main findings are as follows:

Theorem 1: Given that $Z_n = \{1, 2, 3\ldots, n\}$ is a non-negative star-like points, composition of any transformation set $\{\alpha^*, \beta^*, \gamma^*, \ldots\} \subseteq H\omega^*_n$ produces star-like oscillatory sequences for all $x, y, z \in Z_n$ such that $\left|y(\alpha^*\beta^* - z(\alpha^*\beta^*)\right| = |(y\alpha^*)\beta^* - (z\alpha^*)\beta^*|$ and $y \circ (\alpha^* \circ \beta^*)z = y(\alpha^*(\beta^*(z)))$.

Proof:

We need to show that

i.

$$|y - z| \geq |yz - zy| \; and \; |y - z| \geq |y\beta^* - z\beta^*|$$

such that,

$$|y - z| \geq |y(\alpha^*\beta^*)| - z(\alpha^*\beta^*)|$$

Now
$$\begin{aligned} \left|y(\alpha^*\beta^* - z(\alpha^*\beta^*)\right| &= |(y\alpha^*)\beta^* - (z\alpha^*)\beta^*| \\ &\leq |(\alpha^*\alpha^*) - (z\alpha^*)| \\ &\leq |y - z| \end{aligned}$$

Therefore for $\alpha^*, \beta^*, \gamma^* \subseteq H\omega^*_n$.

ii. Let z_i be any star-like point in Z_n, let $\Delta_n(z)$ be the oscillatory order of all star-like transformations of $H\omega^*_n$, then the operation defined on $H\omega^*_n$ forms the composition of the star-like partial transformation. Consider $\Delta_n(z)$ to be associative, we need to show that

$$(\beta^* \circ \alpha^*) \circ z_i = z_j \circ (\gamma^* \circ \beta^*) = z_k \circ (\alpha^* \circ \gamma^*),$$

for any $\alpha^*, \beta^*, \gamma^* \subseteq H\omega^*_n$, such that the following statements are true:

a. $Let\ \alpha^*, \beta^*, and\ \gamma^*$ be three star-like transformations, the composition $(z_i \circ \alpha^*)$ is defined such that $z_i \in \Delta_n(z)^{-1}(\alpha^*) \subseteq H\omega^*_n$.

b. Given that $(y \circ \alpha^*) \circ \beta^* = (y \circ \alpha^*) \circ \beta^*$, then we observed that

$$y\colon (y \circ \alpha^*) \circ \beta^*(z) = y((\alpha^*\beta^*(z))$$

provided that $z_j = \alpha^*\left(\beta^*\left(\Delta_n(z)\right)\right)$ and $z_k \circ (\alpha^* \circ \beta^*)\left(\Delta_n(z)\right) = z_i(\alpha^*\left(\beta^*\left(\Delta_n(z)\right)\right))$.

Under the same conditions in Equation (3).

Theorem 2: Let $G^{\alpha*}$, H^{β^*}, *and* V^{γ} edges, faces and vertices of any star-like harmonic transformation $\alpha^*, \beta^*, \gamma^* \in H\omega_n^*$. If $Dom\left(\alpha^*{}_i, \beta^*{}_j, \gamma^*{}_k\right) \leq Im\left(\alpha^*{}_i, \beta^*{}_j, \gamma^*{}_k\right)$, the decomposition of $\alpha^*, \beta^*, \gamma^*$ yields a star-like disk point $\frac{2^3}{4}e^{u_0}$ such that $|H\omega_n^*| = \frac{1}{2}V^{\gamma^*} = \frac{G^{\alpha^*}-H^{\beta^*}}{2}4\alpha^*\beta^*$.

Proof:

Suppose $H\omega_n^*$ is a starlike harmonic oscillator transformation with a composite relation:

$$a_i^* + b_j^* + c_k^* = H\omega_n^*, \tag{5}$$

Such that for any star-like harmonic oscillatory order $\Delta_n(z)$,

$$H\omega_n^* = \begin{pmatrix} z_n & y_n & z_{n+1} & y_{n+1}\ldots & zy_n \\ \gamma^*Hz_n & \gamma^*Hy_{n+1} & \gamma^*Hz_{n+1} & \gamma^*Hy_{n+1}\ldots & \gamma^*Hz_ny_n \end{pmatrix}.$$

By Definition 3-5 and using system of equation generated from Equation (5), there exist a star-like disk point $\left[\frac{2^3}{4}e^{u_0}\right]$ with $H^{\beta^*} + V^{\gamma^*} = G^{\alpha^*} + |H\omega_n^*| - e^{u_0}\frac{2^3}{4}$.

Then, $\Delta_n(z)|H\omega_n^*| = |\alpha^* Z_{(n+1)} - Y_n| \leq |\alpha^* Y_{(n+1)} - Z_n|$, where

$$\left[\frac{2^3}{4}e^{u_0}\right]V^{\gamma^*} = \Delta_n(z) \tag{6}$$

yields $2^3 e^{u_o} V^{\gamma^*} = \frac{G^{\alpha^*}-H^{\beta^*}}{2}4\alpha^*\beta^* + e^{u_0-1}$,

with $V^{\gamma^*} = e^{u_0-1} + G^{\alpha^*} - 2H^{\beta^*}$.

Therefore, Equation (6) produces the star-like vertices of any $\alpha^*, \beta^*, \gamma^* \in H\omega_n^*$ transformation of harmonic oscillatory order $\Delta_n(z)$, whenever $\frac{1}{2}V^{\gamma^*} = \frac{G^{\alpha^*}-H^{\beta*}}{2}4\alpha^*\beta^*$.

Theorem 3: Let α_1^*, $\alpha_2^* : H\omega_n^* \to P\omega_n^*$, be star-like real valued transformation that are strictly partial star-like harmonic as in Equation (3), then some star-like oscillatory points in Z_n

$z_n = \partial_{\bar{h}}\partial_h\alpha_2^*$, and $y_n = \frac{\left|\partial_{h^2}(\alpha^*{}_2 - \alpha^*{}_1)\right|}{\partial_{\bar{h}}\partial_h\alpha^*{}_1}$ as adopted in Equation (3) produces the star-like oscillatory motions $\alpha_o^*(h) = \frac{1}{2}|h|^2$ and $\alpha^*(h) = \frac{1}{4}(\alpha^*|h|^2 - y(h^2))$, such that $\alpha^*(h) = \alpha_2^*\left(\frac{\wr^* h}{\sqrt{2z_1}}\right) - \left(y_1\frac{\wr^{2*}h^2}{2z_1}\right) = \frac{Z_2}{2z_1}|h|^2 - \left(\frac{(y_1-y_2)\wr^*}{4Z_1}h^2\right)$.

Proof:

Since α_1^*, α_2^* are star-like real valued transformations on $H\omega_n^*$, for a smaller order $n = 5$, we have that

$$\alpha^*(h) = z_n|h|^2 + \mathbb{N}(y_n h^2)$$

where $z_n = \partial\bar{h}\ \partial h\ \alpha_n^*$, $y_n = \partial^2 h\alpha_n^*$.

We observed that there exist finitely many star-like (element) points k, s $\in H\omega_n^*$ when

$$C_k: H_{\alpha^*(h)} \ni u(h) \to ku(kh) \in H_{\alpha^*(kh)}, \tag{7}$$

and

$$w_s: H_{\alpha^*(h)} \ni u(h) \to u(h)e^{sh^2} \in H_{\alpha^*(h)} + \mathbb{N}(sh^2), \tag{8}$$

are perturbed by the harmonic models in Figure (1) and (2) for any star-like real-valued strictly partial harmonic transformation. But when the faces H^*, edges G^*, and vertices V^*converge at a star-like disk point $e^{u_0} = |\wr^*|$ with a 360-degree angular measure, and the transformation satisfies both combinatorial and the oscillatory model using ∂_h to represent the star-like oscillatory points with $|\wr^*| = e^{u_0}$, then, θ^* is the star-like diskpoint from $H_{\alpha_1^*}$ *to* $H_{\alpha_0^*}$ such that

$$\alpha^*(h) = \alpha_2^*\left(\frac{\wr^* h}{\sqrt{2z_1}}\right) - \mathbb{N}\left(y_1\frac{\wr^{2*}h^2}{2z_1}\right)|h|^2 - \mathbb{N}\left(\frac{(y_1-y_2)}{4Z_1}\wr^* \circ h^2\right),$$

Therefore, $\wr^{2*}(y_1 - y_2) = |z_2 - z_1|$ gives

$z\alpha_1^* = y\alpha_{n+1}^*$ and $H\omega_n^* = 360^0$.

Theorem 4: Given that $\theta^* \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, let β^* be chosen as in theorem 2, and let k as in Equation (7), with the oscillatory time (t) then, $|H\omega_n^* 2t|^2 = \frac{1}{4}(2sin4t - 2 + 2cos4t)$.

Proof:

Consider $y_n = \frac{\left|\partial h^2(\alpha_2^* - \alpha_1^*)\right|}{\partial\bar{h}\ \partial h\,\alpha^*{}_1}$, and

$$z_n = \partial\bar{h}\ \partial h\,\alpha_2^*,$$

as in Theorem (3), where $\theta^* = 0$.

The condition that $y_n - Z_n \geq 1$, reduces to $e^{4t} \geq 1$, which agrees with $\Delta_n(z)$ from Equation (6) such that

$$|H\omega_n^*| = \frac{1}{2}V^{\gamma^*} = \frac{G^{\alpha^*}-H^{\beta^*}}{2}4\alpha^*\beta^*, \{at\ t \geq 0\}.$$

But if $\theta^* \neq 0, t = 0$, we observed that

$$y_n - z_n \geq 1, \text{ if } t \in \frac{i\pi}{2}\ \mathbb{N}.$$

But, given that $\theta^* \neq 0, t > 0$, then condition $y_n - z_n \geq 1$ is equivalent to

$$\frac{e^{4t}-1}{|e^{4t}-1|} \leq \frac{sin2t}{|sin2t|} \geq sin|\theta^*|. \tag{9}$$

For any star-like oscillator points in Equation (1).

$$|H\omega_n^* 2t|^2 = (e^t - e^{-t})(e^{\bar{t}} - e^{-\bar{t}}),\ t \neq 0$$

Therefore, when $\theta^* \neq 0$ and $|\theta^*| \leq \frac{\pi}{2}$, where $t \subseteq k$, and that $\frac{(sin2t)^2}{(cos2t)^2} \leq cot|\theta^*|$.

It follows from the computation that

$$\frac{|e^{4t}-1|^2}{(e^{4t}-1)^2} - 1 = \frac{|e^{2t}|^2|sin2t|^2}{e^{4t}(sin2t)^2}$$
$$= \frac{|sin2t|^2-(cos2t)^2}{(cos2t)^2}$$
$$= \frac{(sin2t)^2}{(cos2t)^2}.$$

Hence, $|H\omega_n^* 2t|^2 = \frac{1}{4}(2sin4t - 2 + 2cos4t)$.

Remark: Considering the sequence 1, 6, 30, 32… , we discovered that it has a quadratic recursive oscillatory order $\Delta_n(z) = \sum_{k=0}^{n-1}\binom{n}{n-1-k}\left(n+4-k+\gamma^*(n)\right)$ where $\gamma^*{}_{(n)}$ is a recursive terms of the star-like harmonic oscillator in which $\gamma^*{}_{(1)} = -1, \gamma^*{}_{(2)} = -1,\ \gamma^*{}_{(3)} = -5, \gamma^*{}_{(4)} = 24, \gamma^*{}_{(5)} = 2.$ The recursive terms of the star-like harmonic oscillator of the partial transformation modify the original recursive formula to produce the desired sequence.

5. Conclusion

The research transformed the star-like transformation semigroup $H\omega^*{}_n$ into a visualized star-like oscillatory, resulting in star-like harmonic sequences. These sequences were then applied to the star-like harmonic oscillator recursive relations. The outcome of Theorems 2–4 is based on oscillatory star-like order, which was assumed to be solid, stiff, and have a harmonic oscillator at various axes with a constant star-like angle unless the star-like disk point rotated.

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