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Initial Coefficient Bounds for Some Novel Subclasses of Analytic and Bi-univalent Functions Defined by q-Differential Operator

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Abstract

In this study, we use the q-differential operator to establish two novel subclasses of analytic and bi-univalent functions defined in the open unit disk $D = \{z: z \in \mathbb{C},\ |z| < 1\}$. Some known subclasses of analytic and bi-univalent functions, as well as their results are generalized by these novel subclasses and their results.

1.0 Introduction

Let H represent the class of functions of type

$$f(z) = z + a_2z^2 + a_3z^3 + \cdots \qquad (1.1)$$

which are analytic in the open unit disk $D = \{z: z \in \mathbb{C}, |z < 1|\}$ with the conditions f (0) = 0 and f ′(0) = 1. Also, let S denote the class of functions $f \in H$ that are univalent in D.

The concept of quantum calculus (or q-calculus) was discussed by Ramachandran *et al.* [1] and after its discussion, numerous scholars have worked extensively in this area of study. For instance, [2], [3], [4], [5] defined, investigated and obtained some features for some subclasses of univalent and bi-univalent functions. The q-calculus operator theory is used in many areas of applied sciences such as quantum mechanics, fractional calculus and optimal control.

In [1] and [6] the q-derivative (0<q<1) of functions f ∈ H was defined as

$$\left.\begin{aligned} D_qf(0) &= f'(0) = 1 \\ D_qf(z) &= \frac{f(qz) - f(z)}{z(q-1)} \\ D_q^2f(z) &= D_q\left(D_qf(z)\right). \end{aligned}\right\} \qquad (1.2)$$

From (1.1) and (1.2) we get

$$D_qf(z) = 1 + \sum_{k=2}^{\infty}[k]_qa_kz^{k-1} \qquad (1.3)$$

and

$$zD_q^2f(z) = \sum_{k=2}^{\infty}[k-1]_q[k]_qa_kz^{k-1} \qquad (1.4)$$

where

$$[k]_q = \frac{1-q^k}{1-q}.$$

Note that if $q \to 1$, then $[k]_q = k$.

Every f ∈ S has inverse function f^{-1},
$f^{-1}(f(z)) = z, \quad z \in D,$

$$f\left(f^{-1}(w)\right) = w, \quad w: |w| < r_0f \text{ and } r_0f \geq 0.25,$$

and

$$g(w) = f^{-1}(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - a_2a_3 + a_4)w^4 + \cdots. \quad (1.5)$$

An analytic function f(z) is said to be bi-univalent in D if and only if both f(z) and $f^{-1}(w)$ are univalent in D. Let the class of bi-univalent functions in the unit disk be denoted by *B.* Srivastava and Bansal [7] in their work reported that bi-univalent functions have coefficient bounds $|a_2| <$ 1.51. In 2017, Altinkaya and Yalcin [8] stated that the maximum value of $|a_2| \leq$ 1.33. Brannan and Taha [9] started the study of certain classes of bi-univalent functions class B similar to the familiar subclasses $S^*(\alpha)$ and K(α) of starlike and convex functions of order $\alpha(0 \leq \alpha <$ 1)) respectively. Brannan and Taha [9] showed that a function $f \in H$ is in the class $S_B^*(\alpha)$ of strongly bi-starlike functions of order α (0<α≤1) if

$$\left|arg\left(\frac{zf\prime(z)}{f(z)}\right)\right| < \frac{\alpha\pi}{2} \quad (z \in D, 0 < \alpha \leq 1) \quad (1.6)$$

and

$$\left|arg\left(\frac{wg\prime(w)}{g(w)}\right)\right| < \frac{\alpha\pi}{2} \quad (w \in D, 0 < \alpha \leq 1) \quad (1.7)$$

where g is described by (1.5).

Accordingly, if f ∈ B, then a function f ∈ H belongs to the class $K_B(\alpha)$ of strongly bi-convex

functions of order α (0 < α ≤ 1) if

$$\left|arg\left(1 + \frac{zf\prime\prime(z)}{f\prime(z)}\right)\right| < \frac{\alpha\pi}{2} \quad (z \in D, 0 < \alpha \leq 1) \quad (1.8)$$

and

$$\left|arg\left(1 + \frac{wg\prime\prime(w)}{g\prime(w)}\right)\right| < \frac{\alpha\pi}{2} \quad (w \in D, 0 < \alpha \leq 1). \quad (1.9)$$

Researchers in [8-11] have worked significantly in this field of study and obtained various properties for various subclasses of bi-univalent functions. This study aims to expand the results of several well-known subclasses of analytic and bi-univalent functions by defining two new subclasses using the q-differential operator and obtaining their initial coefficient bounds.

The following lemma is essential for obtaining our results.

Let P be the family of all functions p which are analytic in D and with positive real parts such that $p(0) = 1$.

Lemma 1.1 ([12]). If

$p(z) = 1 + \sum_{k=1}^{\infty} p_k z^k \in P$,

then

$$|p_k| \leq 2, \quad k \in \mathbb{N}.$$

2 Main Results

Definition 2.1. A function $f \in H$ given by (1.1) is in the class $K_B^q(\alpha, s, t, \beta)$ for $0 < \beta \leq 1, \alpha \geq 0, s \neq 0 \; and \; t \neq 0$ if the following inequalities are satisfied.

$$\left|\arg\left[\frac{1-\alpha}{s}\left(s-1+\frac{zD_q f(z)}{f(z)}\right)+\frac{\alpha}{t}\left(t+\frac{D_q(zD_q f(z))}{D_q f(z)}\right)\right]\right| < \frac{\beta\pi}{2} \tag{2.1}$$

and

$$\left|\arg\left[\frac{1-\alpha}{s}\left(s-1+\frac{wD_q g(w)}{g(w)}\right)+\frac{\alpha}{t}\left(t+\frac{D_q(wD_q g(w))}{D_q g(w)}\right)\right]\right| < \frac{\beta\pi}{2} \tag{2.2}$$

where g is given by (1.5).

Remark 2.1

1. As $q \to 1$, the class $K_B^q(\alpha, s, t, \beta)$ reduces to the class $M_B(\alpha, s, t, \beta)$ studied by Krishna et al. [11].

2. For $\alpha = 0$, $s = t = 1$ and $q \to 1$ the class $K_B^q(\alpha, s, t, \beta)$ reduces to class S_B^* studied by Brannan and Taha [9].

Theorem 2.1. Assume that f(z) given by (1.1) belongs to the class $K_B^q(\alpha, s, t, \beta)$ and that $0 < \beta \leq 1, \alpha \geq 0, s \neq 0 \; and \; t \neq 0$. Then

$$|a_2| \leq \frac{2\beta}{\sqrt{2\beta\left[\left(([3]_q-1)-([2]_q-1)\right)\left(\frac{1-\alpha}{s}\right)+([2]_q[3]_q-[2]_q^2)\left(\frac{\alpha}{t}\right)\right]-(\beta-1)\left[([2]_q-1)\left(\frac{1-\alpha}{s}\right)+[2]_q\frac{\alpha}{t}\right]^2}} \tag{2.3}$$

and

$$|a_3| \leq \frac{4\beta^2}{\left[([2]_q-1)\left(\frac{1-\alpha}{s}\right)+[2]_q\frac{\alpha}{t}\right]^2}+\frac{2\beta}{\left[([3]_q-1)\left(\frac{1-\alpha}{s}\right)+[2]_q[3]_q\frac{\alpha}{t}\right]}. \tag{2.4}$$

Proof: (2.1) and (2.2) can be expressed as

$$\left[\frac{1-\alpha}{s}\left(s-1+\frac{zD_q f(z)}{f(z)}\right)+\frac{\alpha}{t}\left(t+\frac{D_q(zD_q f(z))}{D_q f(z)}\right)\right]=[b(z)]^{\beta} \tag{2.5}$$

and

$$\left[\frac{1-\alpha}{s}\left(s-1+\frac{wD_q g(w)}{g(w)}\right)+\frac{\alpha}{t}\left(t+\frac{D_q(wD_q g(w))}{D_q g(w)}\right)\right]=[c(w)]^{\beta} \tag{2.6}$$

respectively, where the functions

$$b(z)=1+b_1 z+b_2 z^2+b_3 z^3+\cdots \in P \tag{2.7}$$

and

$$c(w)=1+c_1 w+c_2 w^2+c_3 w^3+\cdots \in P \tag{2.8}$$

satisfy the inequalities $R\big(b(z)\big)>0 \ \ (z\in D)$ and $R\big(c(w)\big)>0 \ \ (w\in D)$.

Simplifying the LHS of (2.5) gives

$$1+\left[\left([2]_q-1\right)\left(\frac{1-\alpha}{s}\right)+[2]_q\frac{\alpha}{t}\right]a_2 z$$

$$+\left[\left(\left([3]_q-1\right)\left(\tfrac{1-\alpha}{s}\right)+[2]_q[3]_q\frac{\alpha}{t}\right)a_3-\left(\left([2]_q-1\right)\left(\tfrac{1-\alpha}{s}\right)+[2]_q^2\frac{\alpha}{t}\right)a_2^2\right]z^2+\cdots. \tag{2.9}$$

Similarly, simplifying the LHS of (2.6) gives

$$1+\left[\left([2]_q-1\right)\left(\frac{1-\alpha}{s}\right)+[2]_q\frac{\alpha}{t}\right]a_2 w$$

$$+\left[\left(\left([3]_q-1\right)\left(\tfrac{1-\alpha}{s}\right)+[2]_q[3]_q\frac{\alpha}{t}\right)a_3-\left(\left([2]_q-1\right)\left(\tfrac{1-\alpha}{s}\right)+[2]_q^2\frac{\alpha}{t}\right)a_2^2\right]w^2+\cdots. \tag{2.10}$$

Also, the RHS of (2.5) and (2.6) becomes

$$[b(z)]^{\beta}=1+b_1\beta z+\left(b_2\beta+\frac{\beta(\beta-1)b_1^2}{2}\right)z^2+\cdots \tag{2.11}$$

and

$$[c(w)]^{\beta} = 1 + c_1\beta w + \left(c_2\beta + \frac{\beta(\beta-1)c_1^2}{2}\right)w^2 + \cdots, \tag{2.12}$$

respectively.

Furthermore, comparing coefficients in equations (2.9), (2.11), (2.10) and (2.12) gives

$$\left[([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q\frac{\alpha}{t}\right]a_2 = b_1\beta, \tag{2.13}$$

$$\left[([3]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q[3]_q\frac{\alpha}{t}\right]a_3 = b_2\beta$$

$$+\frac{\beta(\beta-1)b_1^2}{2} + \frac{b_1^2\beta^2\left(([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q^2\frac{\alpha}{t}\right)}{\left[([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q\frac{\alpha}{t}\right]^2}, \tag{2.14}$$

$$-\left[([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q\frac{\alpha}{t}\right]a_2 = c_1\beta, \tag{2.15}$$

and

$$-\left[([3]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q[3]_q\frac{\alpha}{t}\right]a_3 = c_2\beta + \frac{\beta(\beta-1)c_1^2}{2}$$

$$-\frac{c_1^2\beta^2}{\left[([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q\frac{\alpha}{t}\right]^2}\left[\left(2([3]_q - 1) - ([2]_q - 1)\right)\left(\frac{1-\alpha}{s}\right)\right.$$

$$\left. + \left(2[2]_q[3]_q - [2]_q^2\frac{\alpha}{t}\right)\right]. \tag{2.16}$$

From (2.13) and (2.15), we get

$$a_2 = \frac{b_1\beta}{\left[([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q\frac{\alpha}{t}\right]} = -\frac{c_1\beta}{\left[([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q\frac{\alpha}{t}\right]} \tag{2.17}$$

which implies

$$b_1 = -c_1. \tag{2.18}$$

Squaring (2.13) and (2.15) and simplifying give

$$2a_2^2\left[([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q\frac{\alpha}{t}\right] = (b_1^2 + c_1^2)\beta^2. \tag{2.19}$$

Also, adding (2.14) and (2.16) gives

$$a_2^2 = \frac{(b_2 + c_2)\beta^2}{\beta[(2[3]_q - 1) - 2([2]_q - 1)]\left(\frac{1-\alpha}{s}\right) + \left[2([2]_q[3]_q - 2[2]_q^2)\frac{\alpha}{t}\right] - (\beta - 1)\left[([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q\frac{\alpha}{t}\right]^2}$$

and utilizing Lemma 1.1, we get the desired result.

Furthermore, subtracting (2.16) from (2.14) gives

$$2\left[([3]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q[3]_q\frac{\alpha}{t}\right]a_3$$

$$= a_2^2\left[2([3]_q - 1)\left(\frac{1-\alpha}{s}\right) + 2[2]_q[3]_q\frac{\alpha}{t}\right] + (b_2 - c_2)\beta + \frac{\beta(\beta-1)}{2}(b_1^2 - c_1^2) \tag{2.20}$$

so that substituting for a_2^2 and noting that $b_1^2 = c_1^2$ from (2.18) gives

$$a_3 = \frac{(b_1^2 + c_1^2)\beta^2}{2\left[([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q\frac{\alpha}{t}\right]^2} + \frac{(b_2 - c_2)\beta}{2\left[([3]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q[3]_q\frac{\alpha}{t}\right]}. \tag{2.21}$$

Applying Lemma 1.1 in (2.21) gives the required result.

Putting s = t = 1 in Theorem 2.1 gives the following corollary.

Corollary 2.2. Let f ∈ H given by (1.1) be in the class $K_B^q(\alpha, \beta)$, $0 < \beta \leq 1$, $\alpha \geq 0$ and

q→1,

then

$$|a_2| \leq \frac{2\beta}{\sqrt{(1+\alpha)(\beta + 1 + \alpha - \alpha\beta)}}$$

and

$$|a_3| \leq \frac{4\beta^2}{(1+\alpha)^2} + \frac{\beta}{(1+2\alpha)}.$$

In Corollary 2.2, we get the following results when we set $\alpha = 0$.

Corollary 2.3. Let $f \in H$ given by (1.1) be in the class $K_B(\beta)$ where $0 < \beta \leq 1$, then

$$|a_2| \leq \frac{2\beta}{\sqrt{(1+\beta)}}$$

and

$$|a_3| \leq \beta(1+4\beta).$$

Definition 2.4. According to (1.1) a function $f \in H$ is considered to be a member of the class $N_B^q(\alpha, s, t, \mu), 0 \leq \mu < 1, \alpha \geq 0, s \neq 0, t \neq 0$ and $z \in D$ if the subsequent inequalities are met.

$$\mathrm{Re}\left[\frac{1-\alpha}{s}\left(s - 1 + \frac{zD_q f(z)}{f(z)}\right) + \frac{\alpha}{t}\left(t + \frac{D_q(zD_q f(z))}{D_q f(z)}\right)\right] > \mu \qquad (2.22)$$

and

$$\mathrm{Re}\left[\frac{1-\alpha}{s}\left(s - 1 + \frac{wD_q g(w)}{g(w)}\right) + \frac{\alpha}{t}\left(t + \frac{D_q\left(wD_q g(w)\right)}{D_q g(w)}\right)\right] > \mu \qquad (2.23)$$

where g is given in (1.5).

Remark 2.5

1. As $q \to 1$, the class $N_B^q(\alpha, s, t, \mu)$ becomes the class $M_B(\alpha, s, t, \mu)$ studied by Krishna *et al*. [11].

2. For $\alpha = 0, s = t = 1$ and $q \to 1$ the class $N_B^q(\alpha, s, t, \mu)$ reduces to class S_B^* studied by Brannan and Taha [9].

Theorem 2.6. Let f (z) given by (1.1) be in the class $N_B^q(\alpha, s, t, \mu), 0 \leq \mu < 1, \alpha \geq 0, s \neq 0, t \neq 0.$ Then

$$|a_2| \leq \sqrt{\frac{2(1-\mu)}{\left[([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q \frac{\alpha}{t}\right]}} \tag{2.24}$$

and

$$|a_3| \leq \frac{4(1-\mu)^2}{\left[([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q \frac{\alpha}{t}\right]^2} + \frac{2(1-\mu)}{\left[([3]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q[3]_q \frac{\alpha}{t}\right]}. \tag{2.25}$$

Proof: There exist $b(z)$ and $c(w) \in P$ from (2.7) and (2.8) such that

$$\left[\frac{1-\alpha}{s}\left(s - 1 + \frac{zD_q f(z)}{f(z)}\right) + \frac{\alpha}{t}\left(t + \frac{D_q\left(zD_q f(z)\right)}{D_q f(z)}\right)\right] = \mu + (1-\mu)b(z) \tag{2.26}$$

and

$$\left[\frac{1-\alpha}{s}\left(s - 1 + \frac{wD_q g(w)}{g(w)}\right) + \frac{\alpha}{t}\left(t + \frac{D_q\left(wD_q g(w)\right)}{D_q g(w)}\right)\right] = \mu + (1-\mu)c(w) \tag{2.27}$$

where $b(z)$ and $c(w)$ have the forms (2.7) and (2.8).

Comparing coefficients of (2.26) and (2.27), we obtain

$$\left[([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q \frac{\alpha}{t}\right] a_2 = b_1(1-\mu), \tag{2.28}$$

$$\left[([3]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q[3]_q \frac{\alpha}{t}\right] a_3 = b_2(1-\mu)$$

$$+ \frac{b_1^2(1-\mu)^2\left(([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q^2 \frac{\alpha}{t}\right)}{\left[([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q \frac{\alpha}{t}\right]^2}, \tag{2.29}$$

$$-\left[([2]_q - 1)\left(\frac{1-\alpha}{s}\right) + [2]_q \frac{\alpha}{t}\right] a_2 = c_1(1-\mu), \tag{2.30}$$

and

$$-\left[\left([3]_q-1\right)\left(\frac{1-\alpha}{s}\right)+[2]_q[3]_q\frac{\alpha}{t}\right]a_3=c_2(1-\mu)$$

$$-\frac{c_1^2(1-\mu)^2}{\left[\left([2]_q-1\right)\left(\frac{1-\alpha}{s}\right)+[2]_q\frac{\alpha}{t}\right]^2}\left[\left(2\left([3]_q-1\right)-\left([2]_q-1\right)\right)\left(\frac{1-\alpha}{s}\right)+\left(2[2]_q[3]_q-[2]_q^2\frac{\alpha}{t}\right)\right]. \quad (2.31)$$

From (2.28) and (2.30) we get

$$a_2=\frac{b_1(1-\mu)}{\left[\left([2]_q-1\right)\left(\frac{1-\alpha}{s}\right)+[2]_q\frac{\alpha}{t}\right]}=-\frac{c_1(1-\mu)}{\left[\left([2]_q-1\right)\left(\frac{1-\alpha}{s}\right)+[2]_q\frac{\alpha}{t}\right]} \quad (2.32)$$

which implies

$b_1=-c_1.$ (2.33)

Also, simplifying (2.28) and (2.30) gives

$$2a_2^2\left[\left([2]_q-1\right)\left(\frac{1-\alpha}{s}\right)+[2]_q\frac{\alpha}{t}\right]=(b_1^2+c_1^2)(1-\mu)^2. \quad (2.34)$$

Addition of (2.29) and (2.31) and simplifying further give

$$a_2^2=\frac{(b_2+c_2)(1-\mu)}{\left[\left([2]_q-1\right)\left(\frac{1-\alpha}{s}\right)+[2]_q\frac{\alpha}{t}\right]} \quad (2.35)$$

and using Lemma 1.1 in (2.35) gives (2.24).

Subtracting (2.32) from (2.30) and substituting for a_2^2 from (2.35) give

$$a_3=\frac{(b_1^2+c_1^2)(1-\mu)^2}{2\left[\left([2]_q-1\right)\left(\frac{1-\alpha}{s}\right)+[2]_q\frac{\alpha}{t}\right]^2}+\frac{(b_2-c_2)(1-\mu)}{2\left[\left([3]_q-1\right)\left(\frac{1-\alpha}{s}\right)+[2]_q[3]_q\frac{\alpha}{t}\right]} \quad (2.36)$$

and utilizing Lemma 1.1 in (2.36) gives the desired result.

Putting $s=t=1$ in Theorem 2.6 gives the following corollary.

Corollary 2.7. Let f ∈ H given by (1.1) be in the class $N_B^q(\mu,\alpha)$ for $0\le\mu<1,\alpha\ge0$,

and q→1, then

$$|a_2| \leq \sqrt{\frac{2(1-\mu)}{1+\alpha}}$$

and

$$|a_3| \leq \frac{4(1-\mu)^2}{(1+\mu)^2} + \frac{(1-\mu)}{(1+\mu)}.$$

From Corollary 2.7 with $\alpha = 0$, we get the following results.

Corollary 2.8. Suppose $f \in H$, as indicated by (1.1) belongs to the class $N_B^q(\mu)$

where $0 \leq \mu < 1$ and $q \to 1$,

then

$$|a_2| \leq \sqrt{2(1-\mu)}$$

and

$$|a_3| \leq 4(1-\mu)^2 + (1-\mu).$$

3.0 Conclusion

Recently, the investigations of the concept of q-calculus got the attractions of many researchers due to its applications in many diverse areas of mathematics and other sciences. In this paper, we defined two new subclasses of analytic and bi-univalent functions in connection with a q-differential operator and we obtained some initial coefficient bounds.

Declarations

Ethics approval and consent

Not applicable

Consent for publication

Not applicable

Competing Interests

The authors declare that they have no competing interest.

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Author's Contributions

RO defined the class $K_B^q(\alpha, s, t, \beta)$, obtained the second and third coefficient bounds and did the general coordination. RA investigated the class $N_B^q(\alpha, s, t, \beta)$ and obtained the property $|a_2|$. AD provided the comprehensive literature review. TO worked on $|a_3|$ for the second investigated class. MO

provided the introduction, references and did the typesetting.

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