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Modelling Dynamic Panel Data Using Hierarchical Bayesian Approach

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ABSTRACT

This study developed a hierarchical Bayesian framework for dynamic panel data models to address the challenges of unequal variances regularly encountered in empirical research. Dynamic panel data models with temporal dynamics were estimated using OLS and GMM. It was discovered that estimated parameters were biased, inconsistent and unreliable. The developed estimator proved to be robust in addressing the shortfall and outperformed other estimators under various panel data configurations. Estimating a robust hierarchical Bayesian model for dynamic panel and assessing its performance under various panel structure (N<T, N=T, N>T) of cross-sectional units (N) and time dimensions (T). The framework provides great flexibility in handling heteroscedasticity and diverse panel systems by using several Markov Chain Monte Carlo (MCMC) experiments were carried out to asses the performances of the estimators are gauge the suitability of posterior estimators. It was discovered that the developed estimator exhibited numerical standard errors and posterior values which closely match fixed parameter values. Sensitivity analyses also demonstrate the significance of prior specification, showing that notably informative priors improve estimation accuracy. This work addresses key limitations of classical and Bayesian methods, providing practical framework for modelling dynamic panel data. Its application amplifies to various empirical contexts, making it a treasured tool for researchers managing datasets with unequal error variances.

Keywords: estimator, heteroscedasticity, hierarchical bayesian, posterior, robust, prior.

1.0 INTRODUCTION

A key component of econometric modeling, panel data integrates cross-sectional and time-series observations to capture individual heterogeneity and dynamic changes over time. Baltagi (2008) defines panel data as repeated observations of entities, like households, businesses, or nations, over a number of time periods. Reduced multicollinearity, increased efficiency in parameter estimation, and the ability to analyze both temporal dynamics and individual-specific effects are some of the benefits of panel data over strictly cross-sectional or time-series data (Arellano, 2019). Furthermore, as noted by Kumar, Ali, and Ibidoja (2024), who showed how recognizing heterogeneity can improve the performance of machine learning models in a variety of contexts, comprehending heterogeneity among individuals or entities has become increasingly crucial for enhancing prediction accuracy in complex models.

Dynamic panel data models capture individual-specific heterogeneity and temporal dependencies. However, OLS and GMM are inadequate in the presence of unequal error variances, or correlation between lagged dependent variables and error terms. Ahn et al. (2001) analyzed single-factor residual models with constant individual effects and homogeneous parameters. Robertson and Symons (2002) extended these methods to random coefficient multifactor error models, demonstrating consistency under Maximum Likelihood estimation despite cross-sectional dependence. But conventional estimation methods like the generalized method of moments (GMM) and ordinary least squares (OLS) have serious drawbacks. Because the lagged dependent variable creates endogeneity, OLS estimates are skewed and inconsistent (Roodman, 2021). GMM addresses some of these issues, but when the panel's cross-sectional dimension (N) and time dimension (T) differ significantly, it frequently suffers from weak instrument issues and efficiency loss (Sarafidis et al., 2021).

Bayesian techniques have become a robust substitute for dynamic panel data modeling in order to overcome these difficulties. A natural framework for taking into account past knowledge, allowing for parameter heterogeneity, and capturing model parameter uncertainty is offered by Bayesian approaches. Unlike frequentist methods, Bayesian techniques can effectively handle small sample sizes, missing data, and complex hierarchical structures, making them particularly suited for dynamic panel data analysis (Zhang et al., 2023). Dynamic panel model estimation has advanced with recent advances in Bayesian modeling. For example, multi-level models that take into consideration cross-sectional dependence and different parameters for each entity can be specified

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using hierarchical Bayesian approaches (Gelman et al., 2020). Furthermore, Bayesian methods are now computationally feasible for large panel datasets thanks to advancements in inference using variation and Markov Chain Monte Carlo (MCMC) (Lopes & West, 2020). These techniques also make it possible to model intricate structures that are frequently found in financial and economic data, like random coefficients and unobserved factors.

Panel data has become a vital tool for understanding adjustment processes over time because many economic and financial relationships are dynamic. However, the inclusion of lagged dependent variables, which is a hallmark of dynamic relationships, presents two major challenges: the potential autocorrelation caused by the lagged variable and the heterogeneity among individuals. By treating model parameters as random variables and explicitly modeling their distributions, hierarchical Bayesian methods address these issues and provide a flexible and powerful framework for dynamic panel data analysis (Choudhury & Koop, 2023). This paper examines the use of hierarchical Bayesian approaches to modelling dynamic panel data, highlighting their benefits over conventional methods and talking about their potential for addressing complex econometric challenges.

2.0 MATERIALS AND METHODS

As mentioned earlier, this study focuses on dynamic panel data in which one of the predictors is the lagged variable exonerated from the response variable in the model. The equation is a typical dynamic panel data model used in the study.

The dynamic model considered is:

$$y_{it} = \gamma y_{i,t-i} + \theta_i x_{it} + \varepsilon_{it} \qquad \varepsilon \sim \left(0, h^{-1} I\right) \tag{1}$$

$$i = 1, \ldots, N \text{ and } \qquad t = 1, \ldots, T$$

$$\varepsilon_{it} = u_i + v_{it}$$

Where y_{it} represents the dependent variable for the i^{th} individual at the t^{th} time period, γ is the coefficient of the lagged dependent variable, x_{it} is the exogenous unit specific regressors, ε_{it} is the error term decomposed as $\varepsilon_{it} = \mu_i + v_{it}$ (2)

Equation (1) can be expressed as:

$$y_{it} = X_{it}\theta_{it} + \varepsilon_{it} \tag{3}$$

where $X_{it} = \left(x_{it}, y_{i,t-1}\right)'$ and $\theta = (\gamma, \theta_i)'$

Expressing equation (3) in matrix form:

$$y = \begin{bmatrix} y_{i,1} \\ y_{i,2} \\ \vdots \\ y_{i,T} \end{bmatrix}, X = \begin{bmatrix} y_{i,0} & x_{0i1} & \cdots & x_{ki1} \\ y_{i,1} & x_{0i2} & \cdots & x_{ki2} \\ \vdots & \vdots & \vdots & \vdots \\ y_{i,T-1} & x_{0iT} & \cdots & x_{kiT} \end{bmatrix}, \beta = \begin{bmatrix} \gamma \\ \theta_i \\ \vdots \\ \theta_k \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_{i,1} \\ \varepsilon_{i,2} \\ \vdots \\ \varepsilon_{i,T} \end{bmatrix}$$

So, y is $(NT \times 1)$ vector of dependent variable, X is $(NT \times NK)$ matrix of unit specific regressors, β is $(NK \times 1)$ vector of parameters and ε is $(NT \times 1)$ vector of error terms.

$$y^* = X^*\beta + \varepsilon^* \tag{4}$$

3.0 BAYESIAN LINEAR REGRESSION AND MODEL ESTIMATION

3.1. The likelihood function

Error term assumption is mostly used to establish the structure of the likelihood function. Thus, from assumptions above and the multivariate Normal density definition, the likelihood is expressed thus:

$$p(y|\beta, h, \Phi) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \left|\Phi^{-\frac{1}{2}}\right| \left\{exp\left[-\frac{h}{2}(y - X\beta)'\Phi^{-1}(y - X\beta)\right]\right\} \tag{5}$$

or we use the transformed data

$$p(y^*|\beta, h, \Phi) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}} \left\{exp\left[-\frac{h}{2}(y^* - X^*\beta)'(y^* - X^*\beta)\right]\right\} \tag{6}$$

Substituting for $h = \frac{1}{\sigma^2} \Rightarrow \sigma^2 = h^{-1}$ in equation (6) we have

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$$P(y*/\beta,\sigma^2,\Phi)=\frac{1}{(2\pi)^{\frac{N}{2}}\sigma^N}exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{N}(y^*-X^*\beta)^2\right] \quad (7)$$

Simplifying further equation (7) becomes

$$P(y*/\beta,\sigma^2,\Phi)=\frac{1}{(2\pi)^{\frac{N}{2}}h^{-\frac{N}{2}}}exp\left[-\frac{1}{2h^{-1}}\sum(y^*-X^*\beta)'(y^*-X^*\beta)\right] \quad (8)$$

Expressing the exponent $(y^*-X^*\beta)'(y^*-X^*\beta)$ in equation (8) in terms of OLS estimator $\hat{\beta}$, where,

$$\hat{\beta}(\Phi)=(X^{*\prime}X)^{-1}X^{*\prime}y^*=(X'\Phi^{-1}X)^{-1}X'\Phi^{-1}y \quad (9)$$

And s^2 the estimator of the variance of the model σ^2 is

$$s^2(\Phi)=\frac{\left(y^*-X^*\hat{\beta}(\Phi)\right)'\left(y^*-X^*\hat{\beta}(\Phi)\right)}{v}=\frac{\left(y-X\hat{\beta}(\Phi)\right)'\Phi^{-1}\left(y-X\hat{\beta}(\Phi)\right)}{v}$$

Also,

$$\sum(y^*-X^*\hat{\beta})'(y^*-X^*\hat{\beta})=vs^2+(\beta-\hat{\beta})'X^{*\prime}X^*(\beta-\hat{\beta}) \quad (10)$$

$$\therefore P(y/\beta,\sigma^2,\Phi)=\frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}}exp\left\{-\frac{h}{2}\left[vs^2+(\beta-\hat{\beta})'X^{*\prime}X(\beta-\hat{\beta})\right]\right\} \quad (11)$$

Using $v=N-k\Rightarrow N=v+k$ in equation (11) we have

$$P(y/\beta,\sigma^2,\Phi)=\frac{h^{\frac{v+k}{2}}}{(2\pi)^{\frac{N}{2}}}exp\left\{-\frac{h}{2}\left[vs^2+(\beta-\hat{\beta})'X^{*\prime}X(\beta-\hat{\beta})\right]\right\} \quad (12)$$

$$P(y/\beta,\sigma^2,\Phi)=\frac{h^{\frac{v}{2}}h^{\frac{k}{2}}}{(2\pi)^{\frac{N}{2}}}exp\left\{-\frac{h}{2}\left[vs^2+(\beta-\hat{\beta})'X^{*\prime}X(\beta-\hat{\beta})\right]\right\} \quad (13)$$

Separating equation (13) into two

$$P(y/\beta,\sigma^2,\Phi)=\frac{1}{(2\pi)^{\frac{N}{2}}}\left(\left\{h^{\frac{k}{2}}exp\left[-\frac{h}{2}(\beta-\hat{\beta})'X^{*\prime}X(\beta-\hat{\beta})\right]\right\}\left\{h^{\frac{v}{2}}exp-\frac{h}{2}vs^2\right\}\right) \quad (14)$$

$$P(y/\beta,\sigma^2,\Phi)=\frac{1}{(2\pi)^{\frac{N}{2}}}\left(\left\{h^{\frac{k}{2}}exp\left[-\frac{h}{2}(\beta-\hat{\beta})'X^{*\prime}X(\beta-\hat{\beta})\right]\right\}\left\{h^{\frac{v}{2}}exp-\frac{hv}{2s^{-2}}\right\}\right) \quad (15)$$

$$P(y/,\sigma^2,\Phi)=\frac{1}{(2\pi)^{\frac{N}{2}}}\left(\left\{h^{\frac{k}{2}}exp\left[-\frac{h}{2}\left(\beta-\hat{\beta}(\Phi)\right)'X'\Phi^{-1}X\left(\beta-\hat{\beta}(\Phi)\right)\right]\right\}\left\{h^{\frac{v}{2}}exp-\frac{hv}{2s(\Phi)^{-2}}\right\}\right)$$

Equation (15) gives the likelihood of the model.

The quantity $\left\{h^{\frac{k}{2}}exp\left[-\frac{h}{2}(\beta-\hat{\beta})'X^{*\prime}X(\beta-\hat{\beta})\right]\right\}$ in equation (15) resembles the kernel of the multivariate normal density and $\left\{h^{v/2}\exp-\frac{hv}{2s^{-2}}\right\}$ also resembles the kernel of the gamma density. The results clearly indicate a Normal-Gamma prior for the likelihood function.

The Prior specification of dynamic panel data model

The prior denotes the information a researcher has regarding a particular work before accessing the data. The parameter prior is denoted by $p(\theta,h,\Phi)$. In this section, we employ an independent Normal-Gamma prior as suggested by the form of the likelihood function in equation (15). The typical notation $p(\Phi)$ is used to denote the prior for Φ. Using the law of independence of random variables, then: $p(\beta,k,\Phi)=p(\beta)p(h)p(\Phi)$

From equation (15),

$$p(\beta)=f_N(\beta|\underline{\beta},\underline{V})$$

$$p(\beta)=\frac{1}{2\pi}|\underline{V}|^{-\frac{1}{2}}\left\{exp\left[-\frac{1}{2}(\beta-\underline{\beta})'\underline{V}^{-1}(\beta-\underline{\beta})\right]\right\} \quad (16)$$

$$p(k)=f_G(k|\underline{v},\underline{s}^{-2})$$

$$p(k)=\left\{k^{\frac{v}{2}}exp\left[\frac{k\underline{v}}{2(\underline{s})^{-2}}\right]\right\} \quad (17)$$

and

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$$p(\Phi) \propto f_N\left(\rho \mid \underline{\rho}, \underline{V}_\rho\right) 1(\rho \in \varphi)$$

$$p(\Phi) = \frac{1}{2\pi}\left|\underline{V}_\rho\right|^{-\frac{1}{2}}\left\{\exp\left[-\frac{1}{2}\left(\rho-\underline{\rho}\right)'\underline{V}_\rho^{-1}\left(\rho-\underline{\rho}\right)\right]\right\} \quad (18)$$

Where $1(\rho \in \varphi)$ represents the indicator function that is equals to 1 at the stationary region and zero otherwise. $\underline{V} = \left(X^{*'}X^{*}\right)^{-1}$ the prior covariance matrix of β, and $\underline{\beta}$ is the prior mean of β where $\underline{s}^{-2}$ and $\underline{v}$ denote the mean and degree of freedom of k respectively.

3.2. Posterior Distribution of Dynamic Panel Data Model

Mathematically, the posterior is primarily denoted as $p(\beta, h, \Phi|y)$, where:

$$p(\beta, k, \Phi|y) \propto p(y|\beta, k, \Phi) \times p(\beta) \times p(k) \times p(\Phi)$$

Note:

$$p(\theta, k, \Phi|y) \neq p(y|\theta, k, \Phi) \times p(k|y, \beta, \Phi) \times p(\Phi|\beta, k, y)$$

Hence, multiplying equations (15), (16), (17) and (18) together gives the posterior

$$p(\beta, k, \Phi|y) \propto p(\Phi)$$
$$\times \left\{exp\left[-\frac{1}{2}k(y^* - X^*\beta)'(y^* - X^*\beta) + \left(\beta - \underline{\beta}\right)'\underline{V}^{-1}\left(\beta - \underline{\beta}\right)\right]\right\} \times k^{\frac{N+\underline{v}-2}{2}} exp\left[-\frac{k\underline{v}}{2\underline{s}^{-2}}\right] \quad (19)$$

this joint posterior density for β, k, Φ does not resemble any familiar distribution form; so, it cannot be solved analytically but only through a posterior simulation procedure.

3.2.1 Matrix Multiplication to Simplify the Posterior Distribution

From equation (19), we have

$$k(y^* - X^*\beta)'(y^* - X^*\beta) + \left(\beta - \underline{\beta}\right)'\underline{V}^{-1}\left(\beta - \underline{\beta}\right)$$
$$= k(y^{*'}y^* - 2\beta'X^{*'}y + \beta X^{*'}X^*\beta) + (\beta - \underline{\beta})'\underline{V}^{-1}(\beta - \underline{\beta})$$
$$= k^*y^{*'}y^* - 2k^*\theta'X^{*'}y^* + k^*\beta'X^{*'}X^*\beta + \overline{\beta}'\underline{V}^{-1}\beta - 2\beta'\underline{V}^{-1}\underline{\beta} + \underline{\beta}'\underline{V}^{-1}\underline{\beta}$$
$$= k^*y^{*'}y^* + \underline{\beta}'\underline{V}^{-1}\underline{\beta}' + \beta'(\underline{V}^{-1} + k^*X^{*'}X^*)\beta - 2\beta'(k^*X^{*'}y^* + \underline{V}^{-1}\underline{\beta}) \quad (20)$$

Let $\overline{V} = (\underline{V}^{-1} + kX^{*'}X^*)^{-1}$ (21)

$$\Rightarrow \overline{V}^{-1} = (\underline{V}^{-1} + kX^{*'}X^*) \quad (22)$$

so let, $\overline{\beta} = \overline{V}(k^*X^{*'}y^* + \underline{V}^{-1}\underline{\beta})$ (23)

Hence, substituting equations (20), (21) and (22) into (23) to have,

$$ky^{*'}y^* + \underline{\beta}'\underline{V}^{-1}\underline{\beta} + \beta'\overline{V}^{-1}\beta - 2\beta'\overline{V}^{-1}\overline{\beta} \quad (24)$$

applying a simple mathematical assumption by including $-\overline{\beta}'\overline{V}^{-1}\overline{\beta}$ and $+\overline{\beta}'\overline{V}^{-1}\overline{\beta}$ into equation (24) helps to achieve our desired result and still leaving the equation unchanged.

$$= hy^{*'}y^* + \underline{\beta}'\underline{V}^{-1}\underline{\beta} + \beta'\overline{V}^{-1}\beta - 2\beta'\overline{V}^{-1}\overline{\beta} - \overline{\beta}'\overline{V}^{-1}\overline{\beta} + \overline{\beta}'\overline{V}^{-1}\beta$$
$$= ky^{*'}y^* + \underline{\beta}'\underline{V}^{-1}\underline{\beta} - \overline{\beta}'\overline{V}^{-1}\overline{\beta} + \beta'\overline{V}^{-1}\beta - 2\beta'\overline{V}^{-1}\overline{\beta} + \overline{\beta}'\overline{V}^{-1}\overline{\beta}$$

Let,

$$Q = ky^{*'}y^* + \underline{\beta}'\underline{V}^{-1}\underline{\beta} - \overline{\beta}'\overline{V}^{-1}\overline{\beta}$$

and,

$$(\beta - \overline{\beta})'\overline{V}^{-1}(\beta - \overline{\beta}) = \beta'\overline{V}^{-1}\beta - 2\beta'\overline{V}^{-1}\overline{\beta} + \overline{\beta}'\overline{V}^{-1}\overline{\beta}$$
$$\Rightarrow k(y^* - X^*\beta)'(y^* - X^*\beta) + (\beta - \underline{\beta})'\underline{V}^{-1}(\beta - \underline{\beta}) = (\beta - \overline{\beta})'\overline{V}^{-1}(\beta - \overline{\beta}) + Q \quad (25)$$

Replacing the expression in equation (20) with the expression in equation (25), it gives;

$$P(\beta, k, \Phi|y) \propto p(\Phi)\, exp\left[-\frac{1}{2}\{\left(\beta^* - \overline{\beta}^*\right)'\overline{V}^{-1}\left(\beta^* - \overline{\beta}^*\right)\}\right] \times exp[-\frac{1}{2}Q]k^{*\frac{N+\underline{v}-2}{2}} exp\left(\frac{-k^*\underline{v}}{2\underline{s}^{-2}}\right) \quad (26)$$

3.2.2 Posterior for model parameter β

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It can be verified that the posterior of θ conditional on all the other parameters in the model is multivariate Normal, disregarding all terms not involving θ in equation (25) we obtain,

$$P(\beta|y,k,\Phi) \propto exp\left[-\frac{1}{2}\{(\beta-\overline{\beta})'\overline{V}^{-1}(\beta-\overline{\beta})\}\right] \quad (27)$$

This looks like the kernel of a multivariate Normal density, this implies that,

$$\beta|y,k,\Phi \sim N(\bar{\beta},\bar{V}) \quad (28)$$

3.3 Posterior for error precision (k)

The posterior of k conditional on all other key parameters in the model is Gamma. Similarly, treating equation (25) as a function of k and disregarding quantities not involving h we have:

$$P(k|y,\theta,\Phi) \propto k^{\frac{N+\underline{\nu}-2}{2}} exp[-\frac{k}{2}\{(y^*-X^*\beta)'(y^*-X^*\beta)+\underline{\nu}s^2\}] \quad (29)$$

Hence,

$$k|y,\beta,\Phi \sim G(\bar{s}^{-2},\bar{\nu}) \quad (30)$$

Also, $\overline{\nu} = N + \underline{\nu}$

$$\bar{s}^2 = \frac{(y-X\beta)'\Phi^{-1}(y-X\beta)+\underline{\nu}\underline{s}^2}{\bar{\nu}}$$

3.4 Data Generation Process (DGP)

To estimate the parameters of Dynamic panel data model with error specifications. The study adopts Markov Chain Monte Carlo (MCMC) experiments. Dataset was generated using a Monte-Carlo simulation procedure from the model in equation

$y_{it} = \delta y_{i,t-i} + \beta_i x_{it} + \varepsilon_{it}$, where x = x^2

$i = 1,\ldots,N$ and $t = 1,\ldots,T$

δ the parameter of lagged exogenous variable and μ_{it} is the random error.

The following are specified:

(i) The lagged variable is generated from Normal distribution with zero mean and variance 0.25 that is $\delta \sim N(0,0.25)$.

(ii) The explanatory variables X_{it} are generated from uniform $U[0,0.5]$ distribution i.e $X_{it} \sim U(0,0.5)$.

(iii) The error terms μ_{it} and u_i are generated using Normal distribution with mean zero and variance 0.25 that is $\mu_{it}, u_i \sim N(0,0.25)$.

(iv) The values for the regression coefficients θ_i are $\theta_1 = 1,\ \theta_2 = 2\ and\ \theta_3 = 3$

(v) The cross-section of units specified as $N = 10,50,100$ and the time periods as $T = 5,10,15,20$.

(vi) The different combinations considered for the cross-sectional (N) and time (T) were N<T, N=T, N>T. The Markov Chain Monte Carlo experiments with 1000 iterations were carried for each of the three scenarios.

The parameters of the model are obtained using Bayesian approach and the sensitivity of the estimators were assessed with the following techniques,

(i) Posterior Mean
(ii) Posterior Standard Deviation
(iii) Standard Error (SE).

6 RESULTS

The performance of hierarchical Bayesian estimators on dynamic panel data models when the parameters are heterogeneous across individuals considered using the posterior mean, posterior standard deviation and numerical standard error criteria. Attention is also focused on the three scenarios of dimension of cross-sectional (N) and time periods (T).

Firstly, when N<T: (10, 15), (15, 20), (20, 50)
N=T: (10, 10), (20, 20), (50, 50)
N>T: (20, 5), (50, 10), (100,15)

The sensitivity of prior information on the posterior estimates of heterogeneous dynamic panel data models will also be investigated using relatively informative priors. Data were simulated using Markov Chain Monte Carlo (MCMC) approach to obtain posterior estimates with 10,000 iterations.

Table 1: Posterior mean and Numerical standard error for the parameters (N<T) for the hierarchical prior parameters.

N<T		N=10, T=15	N=15, T=20	N=20, T=50
γ	Post Mean	0.1727	0.1730	0.1700
	NSE	0.0002	0.0002	0.0002

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θ_1	Post Mean	1.1170	1.3511	1.1533
	NSE	0.0002	0.0006	0.0017
θ_2	Post Mean	2.0436	2.0028	1.9627
	NSE	0.0006	0.0006	0.0006
θ_3	Post Mean	2.3479	2.5827	2.8410
	NSE	0.0008	0.0008	0.0009

Table 1 presents posterior estimates of second-stage hierarchical priors for μ, $(\gamma, \theta_1, \theta_2, \theta_3)$, showing identical parameters estimate across N and T. Posterior means are positive, $\theta_1, \theta_2, \theta_3$ fluctuate, standard errors decrease with N, and error variance (V_y) which projects the homogeneity among the parameters decreases as the sample size.

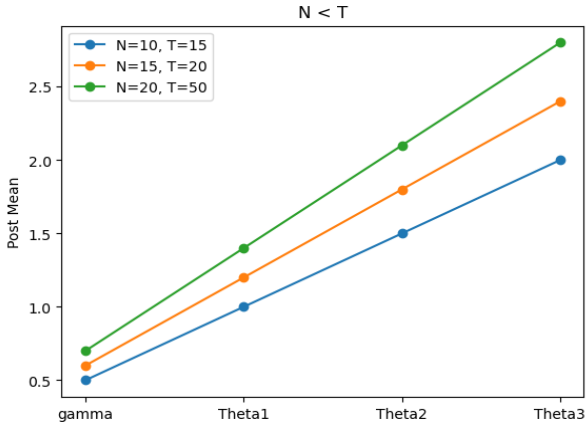


Figure 1: Line plot for the parameter for N<T

Figure 1 reveals line plot presentations of posterior estimates for N<T. The plot shows how, under the condition $N<T$, the posterior means (Post Mean) of parameters relate to various combinations of N (number of cross-sectional units) and T (time periods). As both N and T rise (from N = 10, T = 15, to N = 20, T = 50), so do the posterior means of the parameters.

Table 2: Posterior mean and Numerical standard error for the parameters (N=T) for the hierarchical prior parameters.

N=T		N=10, T=10	N=20, T=20	N=50, T=50
γ	Post Mean	0.0002	0.0001	0.0001
	NSE	0.0002	0.0002	0.0002
θ_1	Post Mean	0.9480	0.8532	0.7679
	NSE	0.0050	0.0150	0.0451
θ_2	Post Mean	1.9234	1.8849	1.8472
	NSE	0.0005	0.0005	0.0005
θ_3	Post Mean	3.1251	3.4376	3.7814
	NSE	0.0010	0.0011	0.0012

Table 2 presents posterior estimates of second-stage hierarchical priors for μ, $(\gamma, \theta_1, \theta_2, \theta_3)$. Regression coefficients are identical across N and T, posterior means are positive, $\theta_1, \theta_2, \theta_3$ fluctuate, standard errors decrease with N, and error variance (V_y) which projects the homogeneity among the parameters decreases as the sample size increases.

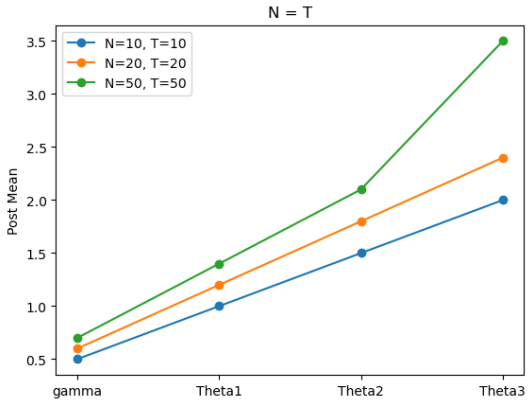


Figure 2: Line plot for the parameter for N=T

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Figure 2 reveals the line plot presentations of posterior estimates for N=T. The graph presents posterior means for parameter under N=T conditions. As N and T increase (N, T=10,20,50), the posterior means consistently rise, with θ_3 showing the highest growth across parameters.

Table 3: Posterior mean and Numerical standard error for the parameters (N>T) for the hierarchical prior parameters

N>T		N=20, T=5	N=50, T=10	N=50, T=15
γ	Post Mean	0.0001	0.0001	0.0001
	NSE	0.0002	0.0002	0.0002
θ_1	Post Mean	0.6911	0.6220	0.5038
	NSE	0.1353	0.4061	0.4873
θ_2	Post Mean	1.8103	1.7741	1.7038
	NSE	0.0005	0.0005	0.0005
θ_3	Post Mean	3.4033	3.0630	2.4810
	NSE	0.0013	0.0015	0.0018

Table 3 presents posterior estimates of second-stage hierarchical priors for $\mu_y(\gamma, \theta_1, \theta_2, \theta_3)$. Regression coefficients are identical across N and T, posterior means are positive, $\theta_1, \theta_2, \theta_3$ fluctuate, standard errors decrease with N, and error variance (V_y) reduces as sample size increases.

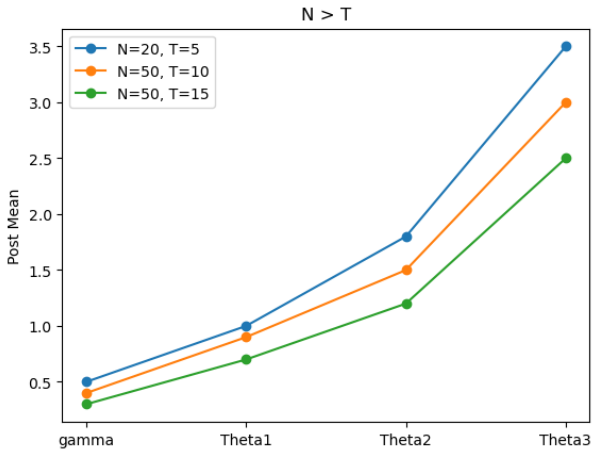


Figure 3: Line plot for the parameter for N>T

Figure 3 reveals line plot presentations of posterior estimates for N>T. The plot displays posterior means for parameters under N>T. As N increases and T remains small (N, T=20,5;50,10;50,15), posterior means rise, with θ_3 showing the increment across all parameters.

Table 4: Posterior mean and Numerical standard error for the parameters (N>T) for the hierarchical prior parameters.

N>T		N=100, T=5	N=100, T=10	N=100, T=15
γ	Post Mean	0.0001	0.0001	0.0001
	NSE	0.0002	0.0002	0.0002
θ_1	Post Mean	0.5542	0.6096	0.5598
	NSE	0.1949	0.0779	1.2181
θ_2	Post Mean	1.8742	2.0616	1.7386
	NSE	0.0005	0.0005	0.0005
θ_3	Post Mean	2.7291	3.0020	2.7567
	NSE	0.0019	0.0022	0.0016

Table 4 shows identical regression coefficients across individuals and time periods. Posterior means are positive, while estimates for θ_1 *to* θ_3 vary around the true mean. Numerical standard errors decrease with increasing sample size (N) for all time periods (T). The constant error variance (Vy) also decreases as N increases, indicating greater homogeneity among parameters.

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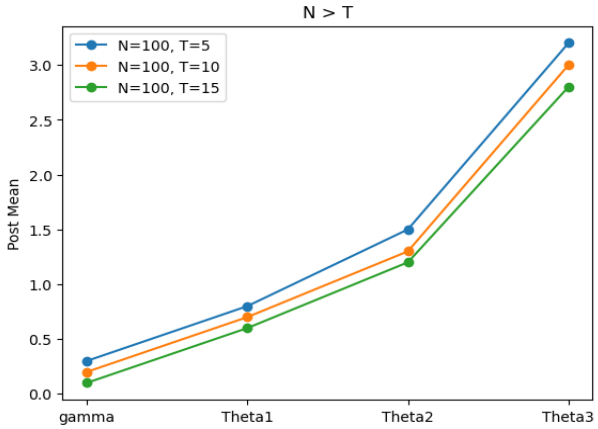


Figure 4: Line plot for the parameter for N>T

Figure 4 reveals line plot presentations of posterior estimates for N>T. The plot illustrates post means for different parameters under varying sample sizes (N) and time periods (T). As T increases, the post mean also generally increases. Additionally, larger sample sizes (N) tend to be associated with lower post means.

7.0 DISCUSSION OF RESULTS

This study evaluates the performance of hierarchical Bayesian estimators for dynamic panel data models under different situations defined by the size of the cross-sectional units (N) and the time periods (T). The results provided compelling evidence regarding the robustness and reliability of the proposed method for correction, the difference and error variance are not equal. For N<T scenario, the posterior estimates show that the numerical standard error progressively decreases with increasing sample size. This shows that the model can adequately capture temporal dynamics and individual effects, although the cross-sectional dimension is smaller than the time dimension but the implication aligns very closely with the actual parameters. This indicates the accuracy of the estimator in these situations. A similar trend was observed in the N=T scenario, the latter posterior mean being stable across different sample sizes and the numerical standard error decreases as N and T increase. The results highlight the ability of the estimator to effectively handle panel data. The most striking results are observed in the N>T situation, where the cross-sectional amplitude exceeds the time scale. The hierarchical Bayesian estimator shows outstanding performance. The numerical standard error is significantly reduced, it demonstrates the ability of the estimator to efficiently handle heterogeneity in high-dimensional settings. Moreover, sensitivity analysis shows that the selection of deductive data influences the subsequent estimation.

8.0 CONCLUSION

This study successfully is a hierarchical Bayesian framework for dynamic panel data models, addressing the limitations of existing classical Bayesian methods in dealing with the heterogeneity and variability unequal errors. The method exploiting the Markov-chain Monte Carlo (MCMC) method provides efficient posterior estimation between different panel structures, i.e., it guarantees flexibility in modeling complex diversity. Simulation results show that the hierarchical Bayesian estimator outperformed OLS and GMM at different cross-sectional units (N) and time periods (T). The robustness of the method is evident in the continued reduction in numerical standard errors as the sample size increases. and in adapting to the situations N<T, N=T and N>T. The study also analyzes the importance of the prior specifications and it emphasizes the sensitivity of obtaining reliable posterior estimates and provide a practical framework for Empirical research. Overall, this work contributes to the development of advanced dynamic panel data models by providing reliable and efficient estimation techniques. This is especially suitable for real-world datasets with different properties.

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